NOTES D'ÉTUDES

ET DE RECHERCHE

COMPETITION AMONG

FINANCIAL INTERMEDIARIES

AND THE RISK OF CONTAGIOUS FAILURES

Olivier De Bandt

September 1994 (revised January 1995)

NER # 30

DIRECTION GÉNÉRALE DES ÉTUDES ET DES RELATIONS INTERNATIONALES

DIRECTION GÉNÉRALE DES ÉTUDES ET DES RELATIONS INTERNATIONALES
DIRECTION DE LA RECHERCHE

COMPETITION AMONG

FINANCIAL INTERMEDIARIES

AND THE RISK OF CONTAGIOUS FAILURES

Olivier De Bandt

September 1994 (revised January 1995)

NER # 30

Competition among financial intermediaries and the risk of contagious failures*

Olivier De Bandt[1]

September 1994 This version: January 1995

*Acknowledgements : The paper is based on the first chapters of the author's dissertation presented to the Department of Economics of the University of Chicago. The author is grateful to all the members of his committee: J. Sdreinknnan, D. Diarrond and J. Cochrane and acknowledges constructive comments from B. Bensaid, B. Biais, C. Kahn, H. Pagès, W. Roberds, J.C. Rochet, P. Sicsic, as well as from participants in seminars at the Bank of France, Chicago, Institut Henri Poincaré, Louvain-la-Neuve (C.O.R.E.), Paris 1, Toulouse (GREMAQ-IDEI). All errors remain his own. The paper does not necessarily reflect the views of the Bank of France.

[1]Correspondence to: Olivier De Bandt. Banque de France. •11-1391 DEER-Centre de Recherche. BP 140-01. 750'19 Paris Cedex 01. France.

Abstract:
The paper presents a model where financial intermediaries invest in a safe and a risky, two-period asset -with aggregate and idiosyncratic shocks on tire risky asset. The realization of returns is privately observed by banks, which offer deposit contracts, with a promised return at t = 1, the level of which depends on the degree of competition in the banking industry. Banks are sensitive .to the propagation of other banks' failures: depositors try to infer the state of the economy.and revise their beliefs after observing too many failures, hence they may watt to rut even on relatively healthy banks (the paper includes a short and a long abstract in French).

Keywords: Asymmetric information, bank runs, systemic risk

JEL classification: D81, D82, G21

Résumé :
Pour formaliser l'existence de faillites bancaires en chaîne on propose un modèle où des intermédiaires financiers peuvent investir dans un actif sîur et un actif risqué. Le rendement de l'actif risqué dépend à la fois de l'état global de l'économie et de caractéristiques propres à chacun des intermédiaires; il n'est connu que par ces derniers. Les intermédiaires proposent des contrats de dépôt en s'engageant auprès des déposants sur la rémuneration versée à t = 1, le niveau de celle-ci dépendant du degré de concurrence dans le système bancaire.
Dans ce cas, les intermédiaires sont sensibles aux faillites des autres banques. En effet, les déposants qui cherchent a déterminer l'état de l'économie, révisent leurs anticipations en observant un grand nombre de faillites de banques, ce qui peut les conduire à retirer massivement leurs dépôts, y compris des banques en bonne santé.

Mots clé : Asymétries d’information, paniques bancaires, risque systémique.

Codes JEL : D81, D82, G21

Résumé Long

Pour établir un lien entre la structure du système bancaire et l'apparition de paniques bancaires, on présente un modèle où les paniques bancaires trouvent leur origine dans une asymétrie d'information entre les intermédiaires financiers et les déposants.

En cela, le modèle s'inspire de Jacklin et Bhaitacharya (1988) et se distingue des formulations à la Diamond et Dybvig (1983) où la panique a une origine extérieure au modèle mais se révèle autoréalisatrice. Ici, au contraire, l'incertitude des déposants quant à l'effet d'un choc sur leur propre banque les conduit a retirer leur dépôts de façon préventive. La généralisation au plan collectif de tels comportements individuels peut déboucher alors sur une panique bancaire. Par ailleurs, le système bancaire n'est pas réduit a une banque unique, mais on modélise explicitement une multiplicité de banques, ce qui permet d'introduire le jeu de la concurrence entre les intermédiaires.

L'idée de base est qu'en raison d'asymétries d'information les intermédiaires financiers sont soumis à des "externalités informationnelles" de la part des autres intermédiaires.

Formellement, le modèle comprend trois périodes (t = 0, 1, 2). En t = 0, les déposants confient leurs avoirs aux intermédiaires qui ont seuls accès à des projets risqués, donnant des revenus en t = 1 et t = 2. Le rendement du projet d'un intermédiaire donné dépend de l'état de l'économie - supposé déterminé une fois pour toutes en t = 1- et d'un facteur idiosyncratique, dont le tirage en t = 1 et t = 2 est effectué de façon indépendante, dans le temps et entre les intermédiaires. En t = 1, seuls les intermédiaires connaissent le rendement de leur projet (en revanche, il y a information parfaite en t = 2). Les intermédiaires proposent des contrats de dépôt aux consommateurs, en leur assurant un rendement fixe r en t = 1, qui est réparti de façon optimale entre les déposants. Si un intermédiaire n'est pas en mesure de respecter ses obligations contractuelles en t = 1, la banque est liquidée. C'est une "faillite fondamentale".

Il convient de résoudre le programme de chaque intermédiaire, compte tenu de la forme prise par la concurrence au sein du système bancaire. Les consommateurs ont de l'aversion pour le risque et sont soumis à un choc de liquidité. Comme dans la plupart des modèles de paniques bancaires (en anglais, "bank ruas"), les consommateurs ignorent en t = 0 s'ils vont ¯être de "type" 1 ou de "type" 2, c'est à dire s'ils tireront relativement plus d'utilité de la consommation en période 1 ou en période 2. Les intermédiaires assurent en t = 0 les consommateurs contre ce choc de liquidité. Toutefois le `type" n'est connu, en t = 1, que par les déposants. Ces derniers se présentent à leur banque et sont autorisés à effectuer des retraits correspondant au `type" annoncé. Les intermédiaires doivent donc définir un schéma d'incitation dans lequel les déposants ont intérêt à révéler leur vrai "type".

Dans un premier temps (en t = 0), on détermine le contrat passé par les banques, c'est-à-dire les retraits autorisés par les deux types de déposants, sous l'hypothèse que ceux-ci ne connaissent que la solvabilité de leur propre banque. Les déposants se répartissent de façon égale entre les différentes banques qui se font concurrence à la Bertrand, si bien que toutes les banques proposent la même allocation et ont à résoudre le même programme qui maximise l'utilité des déposants. L'allocation optimale est telle que les retraits autorisés en période 1 sont relativement plus importants pour les individus de "type" 1 que pour ceux de "type" 2. C'est l'inverse en période 2.

Dans un deuxième temps (en t = 1), les déposants observent les faillites fondamentales dans l'ensemble du système bancaire et révisent leurs anticipations quant au rendement de leurs avoirs en période 2. S'ils observent un trop grand nombre de faillites, ils vont conclure que l'économie se trouve en récession (l'état agrégé est "mauvais"). Les déposants de "type" 2 vont alors préférer l'allocation de "type" 1. C'est une panique bancaire, puisque, dans toutes les banques, les retraits sont supérieurs à ce que les intermédiaires avaient initialement prévu. Durant une panique, seuls certains intermédiaires seront en mesure de satisfaire le surcroît de demande de la part des déposants. Les autres vont faire "faillite par contagion". Ce deuxième type de faillite est causé par l'absence de coordination entre les déposants. Les liquidations qui s'ensuivent ne sont pas socialement optimales, puisqu'en général (sauf si les agents ont très peu d'aversion pour le risque ou si la valeur de liquidation de la banque est très élevée), il serait préférable de continuer l'activité des banques, même si l'économie est. dans le "mauvais" état agrégé. Dans ce cas une "suspension de la convertibilité" -durant laquelle les banques sont autorisées à rationner les retraits des déposants- a pour effet d'assurer un répit aux intermédiaires, mais les anticipations des déposants ne sont pas pour autant modifiées.

Dans la mesure où la valeur de liquidation d'une banque en cas de faillite est plus faible en période de récession prolongée, qu'en phase de croissance, le modèle permet de relativiser les analyses de de Friedman et Schwarz (1963) qui expliquent le nombre considérable de faillites après le "bank holiday" de mars 1933 par le retard dans la décision du Federal Reserve System d'imposer une "suspension de[4] la convertibilité". En effet, à la différence de la situation antérieure à 1913 -où l'intervention des associations locales de banques, les "clearing houses", avait pour fonction, lors d'une suspension, de sélectionner les banques solvables et de fermer les banques déficitaires- la menace d'une "suspension de la convertibilité" entre 1929 et 1933 n'aurait pas conduit les déposants à réviser leur opinion sur la solvabilité du système bancaire du fait de l'ampleur de la récession. Avant comme après 1913, la "suspension de la convertibilité" était, certes, socialement optimale. Toutefois, d'une part, les faillites bancaires entre 1929 et 1933 étaient plus nombreuses et, comme le décrivent Friedman et Schwarz, le système bancaire était tenu en réelle suspicion. D'autre pari, durant la Grande Dépression, les incitations à retirer les dépôts étaient d'autant plus forces qu'à l'occasion d'une liquidation les déposants anticipaient une plus forte décote sur leurs avoirs. Au total, les banques ont fait faillite parce que l'économie était en récession et l'intervention des pouvoirs publics sous la forme d'une «suspension de convertibilité" anticipée aurait été vouée l'échec. D'autres formes d'action auraient été nécessaires. Ceci rejoint d'ailleurs la deuxième thèse de Friedman, à savoir que l'incapacité du Federal Reserve System de l'époque à assurer un refinancement satisfaisant des banques avait accentué la crise.

Dans une dernière partie, le modèle à trois périodes est rendu plus dynamique par la répétition du même jeu un nombre infini de fois. Dans ce cas on montre que d'autres types d'équilibres peuvent apparaître, comme la collusion entre les banques sur le niveau de r. La collusion entraîne la quasidisparition du risque de «faillite fondamentale". Il existe donc bien un arbitrage entre la stabilité et l'efficience du système bancaire, bien que la levée des restrictions à la concentration n'entraîne pas forcément une aggravation du pouvoir de monopole.

Pour conclure, il apparaît que du fait des interactions de nature informationnelle entre intermédiaires financiers, il existe une incitation, pour les banques, d'une part à contrôler la diffusion de l'information aux déposants, et d'autre part, à mettre en place des mécanismes coopératifs d'assistance. Par ailleurs, ce type de modèle peut aussi se révéler utile pour étudier l'effet de la révision des anticipations en situation d'information asymétrique sur le marché interbancaire, sur des marchés de produits dérivés ou pour l'analyse du crédit interentreprise.

1 Introduction

1.1 Relevant issues and review of the literature

The aim of the paper is to investigate the convection between the structure of the banking industry and the existence of a `systemic risk', or a risk of general collapse of the banking system. Economists and bank practioners have always been concerned with the risk of bank panics. In the United States, with the extreme fragmentation of the banking system, panics have often spread out of apparently local or idiosyncratic shocks, leading to a `contagion of fear' (Friedman and Schwarz (1963)). The reality of the problem is evidenced by a priori surprizing behaviors, like the participation of banks to the financing of `rescue packages' in order to bail out failing intermediaries. Helping a potential competitor is usually in contradiction with banks' self interests, but there are many examples, in economic history, of attempts by the banking industry, to avoid failures that may extend to the whole banking system. For instance, Williamson (1989) gives evidence of cooperative behavior among Canadian banks during the period 1870-1913 in case of a run: to prevent spillover effects, the largest banks were willing to act as'informal Lender of Last Resort (L.L.R.) and managed to stop the run by keeping the small banks open. Sirnilarly, in their analysis of the U.S. National Banking System (1863-1914), Friedman and Schwarz (1963) describe the rote of reserve banks and clearing houses in the provision of assistance to banks in trouble [The US Federal Reserve System was created in 1913]. In particular `loan certificates' (joint liability of the members of a clearing house) were issued to avoid monetary contraction and to diffuse the risk of financial panics (see also Timberlake (1984), Corton (1985), as well as Gorton and Mullineaux (1987) on the yole of clearing houses).

A possible explanation, presented in the paper, is that, due to the existence of assymetric information, banks are subject to an externality arising from the intermediation activity of other banks. When banks are buffeted by correlated shocks, depositors infer the state of their particular bank from partial information they receive about the whole banking system. In some cases, when banks cannot credibly reveal the.true state of the economy, depositors may panic and decide to `run' on the banks.(Le. to rush in and withdraw massively their deposits). In those circumstances, banks share a common interest in the stability of the banking system and have an incentive to prevent depositors from knowing `too much' about. the actual state of the banking system.

It is worth noting Chat, beyond the case of the U.S. banking system, a mode] of `runs' provides a convenient framework to address related questions of risk-management in a decentralized financial system. Let us mention, first, the case of derivative markets, where the effect of a large price shock on the financial condition of a counterparty is not public information. In that case, outsiders miglit conclude that troubles at one firm are also present at other firms with similar activity. Consequently an initial shock can spread to other institutions, even with weak links between them. A second example is trade credit, wliere firms extend credit to other firms they do business with. Firms are interested in finding out whether or not the economy is experiencing a recession, but, for a given firin, the observation of its own output is not a good indicator. If many firms are going bankrupt, other firms will revise their beliefs about the likelihood of being in recession. Trade credit lines will therefore be suddenly severed and healthy firms threatened to go bankrupt. Hence the initial fears of recession will be ex post. validated. In both cases, as in the particular case of banks, there is a process of contagion based on information externalities.

Formally, the paper is a contribution to the literature on bank runs, whose models started with the seminal Diamond and Dybvig (1983) paper (hereafter DD). In this setting, depositors' confidence plays a crucial role in the optimality of intermediation. In the paper, to be consistent with actual banking practices, a multiple banking system (and not only a single bank) is introduced. This allows to measure the impact of interbank competition on the stability of the banking system. In fact, the idea of a tradeoff between the degree of competition and the safety of the banking system is not new. It lias influenced significant parts of the U.S. banking legislation, and lias also been an important issue in the European Monetary Union [see Aglietta and Moutot (1993)]. Formally, the link between depositors' confidence and competition was made by Smith (1984, 1992) with the introduction of -a multiple-bank system, then by Aghion, Bolton and Dewatripont (1988) (hereafter ABD), and by Matutes and Vives (1992).

However, in many of these models, the analysis is concentrated on the competition for deposits among intermediaries investing in non risky assets but facing liquidity shocks only, as in DD. On the contrary, Jacklin and Bhattacharya (1988) (hereafter JB) introduced explicitly asymmetric information on the return of a risky asset, but in a one-bank model. Empirically, uncertainty on the asset side is a major source of disturbance in banking activities , even if it is amplified by the behavior of depositors [For a similar view, see Bernanke and Gertler (1987). Calomiris and Gorton (1991) provide historical evidence in favor of the asymmetric information view of bank panics.] Thus the central point of the paper is to introduce a non-diversifiable portfolio risk in a multiple-bank model. With correlated shocks across intermediaries, joint failures are observed and, depending on depositors' expectations, new transmission mechanisms of failures may arise [This effect is independent of the existence of direct participation, as in Mislikin (1992), where a "small" bank lias deposits in a "large" bank, and the withdrawal of funds by the former, in case of failure, extends it to the latter.].

1.2 Main results

In the model developed in the following section, banks invest in a safe and a risky asset and commit *ex ante* to offer a fixed payment on deposits. The realization of returns on risky investments, with aggregate and idiosyncratic shocks, are privately observed by banks. The model extends JB's model by considering first, a multiple-bank system, then, more than two states of the world and, finally, the perception of returns at $t = 1$ and $t = 2$, instead of $t = 2$ only. The perspective is, however, slightly different since intermediation is constrained by the legal environment. The game may also be repeated an infinite nuinber of periods.

A crucial feature of the model is" that the inabifty of some banks to pay the promised return at $t = 1$ is at the origin of `fundamental' bank failures. But failures may also be contagious in the sense that depositors use the information available to revise their beliefs about the state of the economy and may conclude that it is optimal for them to run on the banks. Banks cannot credibly reveal what the true state of the world is, so that, in one equilibrium, depositors may decide to withdraw their deposits in the `good' state of the economy.

This extends ABD's result to shocks on the asset side in a more dynamic framework, with explicit analysis of the effect of competition among banks. The nature of competition within the banking system affects the level of the *ex ante* promised payments to depositors and determines the probability of failure of an individual bank.

1.3 Structure of the paper

The following sections are organized as follows. The basic framework of the model is presented and motivated in section 2. Then, section 3 solves the representative intermediary's problem in the case of perfect competition. Information of depositors about failures of other intermediaries is introduced in section 4 in order to generate `information-based' runs. A dynamic version of the model is presented in section 5. Section 6 discusses the robustness of the model and concludes.

2 The model

The model has three periods: $t = 0, 1, 2$. There are two groups of agents: depositors and intermediaries.

2.1 Preferences

1. There exists a continuum of risk-averse consumers. Liquidity shocks are introduced as in DD. At $t = 0$, each agent faces uncertainty regarding lier type: each consumer may be either an early consumer (`type l'), in proportion p, or a late consumer (`type 2') in proportion 1 - p. Types are privately observed by agents at the beginning of $t = 1$.

Consumption at date i of type j consumer is noted cil and lier utility over date 1 and 2 is given by $U(c_{lj}) + p_j\, U(c_{2j})$, with pl = p, such that $0 < p < 1$; and $p_2 = 1$. This heterogeneity between agents plays a crucial role in the existence of `runs'. To get numerical results, a functional form is assumed for the utility function with $U(c_{ij}) = \sqrt{c_{ij}}$, as in JB.[1]

2. Financial intermediaries are risk neutral and specialize in the monitoring of risky `projects' (to be defined later). Intermediaries are indexed by k (k = 1, ... K). An intermediary who knows that lie is going to fail w.p.1, suspends the activity of his individuel bank. One interpretation is that the bank tries to minimize the cost of remaining open. Suspension implies that the bank's assets are liquidated.[2]

2.2 Technology

1. Agents receive an endowment of 1 unit of the consumption good at period i = 0. A zero cost storage technology is available between period 0 and period 1, yielding 1 unit of the consumption good for each unit invested at t = 0. This can be viewed as a `short' asset at t = 0. To simplify, it is also assumed, as in many models of bank runs, starting from DD, that the storage technology is not available between t = 1 and t = 2. The latter assumption is not crucial, however.[3]

2. Each intermediary may invest in a risky `project' yielding returns at t = 1 and t = 2. There is locally no constraint on the size of the project and there are as many projects as intermediaries. All the projects are ex ante identical. In comparison to the storage technology, this is a `long' asset. More precisely:

• Each intermediary privately observes the return on his `project' at t = 1, but there is perfect information on the returns at t = 2.

• The return to intermediary k on his project, r_t^k , is the result of an aggregate shock and an idiosyncratic shock. There are two realizations of the aggregate shock: the projects are, in average, either good (G) or bad (B), witli probability p_G and 1 - p_G, respectively. Idiosyncratic shocks are independently distributed across intermediaries. This can be formalized by assuIning that returns are drawn from one out of two probability distributions F_A (A = G. B), with the same support [r^-, r^+].

• In addition, to allow inference of the retiens at t = 2 given the realization at t = 1, 1 assume that there is correlation of returns over time. This property is illustrated in a very simple fashion: the aggregate state (or, equivalently, the distribution F_A) is determined, once and for ail, at t = 1. For example, if the aggregate shock is G, the return at date t to intermediary k is *suc11* that $\Pr ob\{r_t^k < y\} = F_G(y)$, for t = 1,2 and we have to impose that F_G first corder stochastically dominates F_B. Namely, for all y belonging to the support of F, $F_G(y) \leq F_B(y)$.

• An intermediary may decide at the end of t = 1 to interrupt his `project'. In that case, he receives a fixed liquidation value of L_υ.

3. Intermediaries offer deposit contracts.

• Deposit contracts are constrained to be of the following form: at t = 0, each intermediary invests the money collected from consumers in the two types of assets available and commits to pay at t = 1 a fixed return $\bar{r}$ on the risky asset, then the `project' is liquidated at the end of t = 2 and depositors receive r_2^k. The design of the contract is intended to mimick actual banking practices: deposits are locked in but yield returns at different points in time and provide insurance against the event of "being an early consumer". Liquidation at t = 2 is designed to introduce an equity participation comportent in the relationship between banks and their depositors.[4]

• It is also assumed that there remains a non-diversifiable portfolio risk. For that, I require that each depositor invests lier whole endowment in one and only one intermediary (of course, intermediaries receive deposits from many depositors) and that ex ante pooling of risky projects across intermediaries is limited. The latter assumption is motivated by competitive reasons or incentive motives if the intermediary can choose among different levels of effort (or projects of different riskiness). The last reason may be found in the political economy literature, as in Economides, Hubbard and Palia (1994) for the U.S. case (small banks have historically been lobbyillg to keep their independence and to avoid being taken over by large banks).

• An intermediary's net return in t = 1 on its investment is therefore: $\begin{cases} r_1^k - \bar{r} & if\ r_1^k \geq \bar{r} \\ 0 & otherwise \end{cases}$

4. The following chronology of events is assumed to hold at t = 1:

• First, depositors learn their type. The aggregate shock A as well as the idiosyncratic shocks I_1^k are drawn, so that returns r_1^k are known, for ail k. Bank k may decide to suspend individually. The information is known costlessly by all the depositors of k. In addition, it is convenient, but not essential, to assume that liquidation takes time and is only organized at the end of $t = 1$.[5]

• Then, depositors can withdraw funds. A depositor announces a type, say j, and she receives C_{lj}, then C_{2j}, at. $t = 2$. There is a Sequential Service Constraint so that depositors stand in fine and are served on a first corne-first served basis.[6]

• Finally, the liquidation value L_v is available at the end of $t = 1$.

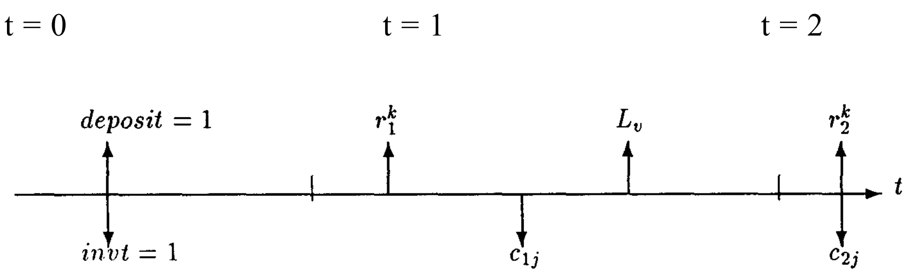

Figure 1: Timing. ↑ are inflows and ↓ are outflows for intermediary k

Summary of timing

• Investment is made at t = 0: intermediary k receives 1 unit of the consumption good and invests it.

• At t = 1, consumers learn their type and intermediary k receives a return r_1^k on his risky asset. Then a depositor, who announces that she is a type j, is allowed to withdraw c_{1j}. A liquidation value of L_v is available at the end of t = 1 if intermediary k does Dot carry the risky project over t=2.

• At t = 2, intermediary k receives a return of r_2^k and depositors withdraw c_{2j}.

3 Case 1: Competition ex ante, no communication ex post

The representative intermediary's problem is to find the optimal level of $\bar{r}$. It is provisionally assumed in this section that depositors in different banks have no information about the solvency of other banks at t = 1, so that the failure of one bank does not entail any revision of beliefs about the aggregate state. This assumption will be relaxed in section 4.

3.1 Optimal announcement

The optimal level of i• is the solution of a 2-stage game, where competition among banks is limited to the level of the promised return $\bar{r}^k$

First, intermediaries solve for the optimal allocation between the 2 types of depositors for each possible level of the promised return. For a given $\bar{r}$ if $r_1^k < \bar{r}$, intermediary k knows that lie is going to be run w.p.l. Consistently with what was assumed in section 2, in corder to avoid a run, intermediary k suspends before the period where depositors are allowed to withdraw their deposits. This will happen with probability $1 - P_{\bar{r}} = F(\bar{r}) = p_G F_G(\bar{r}) + (1 - p_G) F_B(\bar{r})$.The information is perfectly known to k's customers (e.g. they find their bank's doors closed). Otherwise, depositors receive the promised return with probability $P_{\bar{r}}$.

Then, intermediaries compete with each other in Bertrand fashion. As in the standard Bertrand model with two intermediaries k and 1, market shares will depend on $\bar{r}^k$ and $\bar{r}^l$. If $\bar{r}^k > \bar{r}^l$ will attract all depositors and 1 will get none of them. In a one-shot game, k and l will choose the same $\bar{r}$, such that the utility of consumers will be maximized. The same argument can be made with more than two intermediaries. Due to the existence of a continuum of depositors, the proportion of the two types of depositors is the same for each intermediary as at the aggregate level (namely p and 1 - p).

3.2 Optimal level of the promised return

To maximize depositors' expected utility, the intermediary face a tradeoff between the level of the promised return and the probability of failure: the higher the promised return, the lower the probability to be able to pay it.. If r is discrete, the optimal return is one of the realizations of r: if $\bar{r}$ is optimal, the gain in consumer's expected utility from a slightly higher return than $\bar{r}$ is greater or equal to the loss from a higher failure rate.

1. Intermediary k solves the following program (P1), where the superscript k is dropped for $\bar{r}$ and r_t, since all the intermediaries are identical ex ante:

$$\max_{c_{ij},\alpha,\bar{r}} \quad P_{\bar{r}} \quad \left\{p\left[U(c_{11})+\rho E(U(c_{21})|r_1 \geq \bar{r}\right]+(1-p)\left[U(c_{12})+E(U(c_{22})|r_1 \geq \bar{r}\right]\right\}+U_0(\bar{r},L_\upsilon,\alpha) \qquad (1)$$

$$\text{s.t.} \qquad 0 \leq \alpha \leq 1 \qquad (2)$$

$$1 \leq \quad P_{\bar{r}} \quad \left\{p\left[U(c_{11})+\rho E(U(c_{21})|r_1 \geq \bar{r}\right]+(1-p)\left[U(c_{12})+E(U(c_{22})|r_1 \geq \bar{r}\right]\right\}+U_0(\bar{r},L_\upsilon,\alpha) \qquad (3)$$

$$\alpha+(1-\alpha)\bar{r} \geq pc_{11}+(1-p)c_{12} \qquad (4)$$

$$(1-\alpha)r_2 \geq pc_{21}+(1-p)c_{22} \qquad (5)$$

$$U(c_{11})+\rho E(U(c_{21})|\ r_1 \geq \bar{r}) \geq U(c_{12})+\rho E(U(c_{22})|\ r_1 \geq \bar{r}) \qquad (6)$$

$$U(c_{12})+\rho E(U(c_{22})|\ r_1 \geq \bar{r}) \geq U(c_{11})+\rho E(U(c_{21})|\ r_1 \geq \bar{r}) \qquad (7)$$

where $P_{\bar{r}}$ is $\Pr ob\{r_1 \geq \bar{r}\}$, α is the share of the intermediary's portfolio invested in the safe asset and

$$U_0(\bar{r},L_\upsilon,\alpha)E(U(\alpha+[1-\alpha][r_1+L_\upsilon])\ |\ r_1 < \bar{r})(1-P_{\bar{r}}) \qquad (8)$$

is depositor's utility in the case of suspension. To keep the problem interesting, it is reasonable to assume an upper bound on L_υ, so that:

$$U(L_\upsilon) \leq \rho E(U(r_2)) \qquad (9)$$

This means that type 1 depositors prefer the stochastic return at t = 2 to the risk-free liquidation value at t = 1. Automatic liquidation of the risky asset is therefore avoided.[7]

Depositors face borrowing and short-selling constraints (equation (2)). In addition, reservation utility is set. equal to the return from investing the whole portfolio in the safe asset (constraint (3)). (4) and (5) are resource constraints. (6) and (7) are incentive constraints: the allocation lias to be such that each consumer truthfully announces her type. Equations (6) and (7) should include $P_{\bar{r}}$; and U_0, but those two terms cancel out on the LHS and the RHS. Using Lemma 1, presented below, it. can be shown that (7) is not binding.

Since depositors receive the whole return at t = 2, their allocation depends on the realization of r_2.. However, expected utility is conditional on the event $r_1^k \geq \bar{r}$.

To integrate a possible conflict of objectives between a bank and its depositors, an additional constraint should be included, namely that the expected return for intermediaries is positive:

$$(1-\alpha)E(r-\bar{r}\ |\ r \geq \bar{r})\Pr\{r \geq \bar{r}\} \geq 0 \qquad (10)$$

By construction, constraint (10) is always satisfied, since banks cannot give to depositors more than what they have. Problem (P1) is therefore solved by dropping (10).

2. Using the proportionality of the c_{2j}'s to r_2, it is shown in Appendix A1 and A2 that the solution to (P1) can be found by solving the following program (P2) and imposing that depositors receive at $t = 2$ a share r_2/r^+ of the maximal allocation available at $t = 2$ (r_2 is the realization of the return at $t = 2$ and r^+ is the upper bound of the support of r_2):

$$\max_{c_{ij},\alpha,\bar{r}} \quad P_{\bar{r}} \quad \{p[U(c_{11}) + \rho U(c_{21})Q_{\bar{r}}] + (1-p)[U(c_{12}) + U(c_{22})Q_{\bar{r}}]\} + U_0(\bar{r}, L_\upsilon, \alpha) \tag{11}$$

$$\text{s.t.} \qquad 0 \leq \alpha \leq 1 \tag{12}$$

$$1 \leq \quad P_{\bar{r}} \quad \{p[U(c_{11}) + \rho U(c_{21})Q_{\bar{r}}] + (1-p)[U(c_{12}) + U(c_{22})Q_{\bar{r}}]\} + U_0(\bar{r}, L_\upsilon, \alpha) \tag{13}$$

$$\alpha + (1-\alpha)\bar{r} \geq pc_{11} + (1-p)c_{12} \tag{14}$$

$$(1-\alpha)r^+ \geq pc_{21} + (1-p)c_{22} \tag{15}$$

$$U(c_{11}) + \rho U(c_{21})Q_{\bar{r}} \geq U(c_{12}) + \rho U(c_{22})Q_{\bar{r}} \tag{16}$$

$$U(c_{12}) + U(c_{22})Q_{\bar{r}} \geq U(c_{11}) + U(c_{21})Q_{\bar{r}} \tag{17}$$

where r^+ is the upper bound of the support of r and $Q_{\bar{r}}$ is defined as follows:

$$Q_{\bar{r}} = E(\sqrt{\frac{r_2}{r^+}}|r_1 \geq \bar{r}) \tag{18}$$

where the functional (square-root) form of the utility function is explicitly taken into account.[8]

$Q_{\bar{r}} < 1$ is a weighting function, which measures the expected utility of the risky return, normalized *by* r^+−the upper bound of r− given that the intermediary is able to pay at $t = 1$ the promised return.[9]

It is useful, at this stage, to introduce a little bit of notation that will prove useful in the next section.

Since the distribution of r is given by the mixture of two distributions it is convenient to write that:

$$Q_{\bar{r}} = W_G(\bar{r})\Sigma_G + W_B(\bar{r})\Sigma_B \tag{19}$$

where $W_G(\bar{r}) = prob\{G|r_1 \geq \bar{r}\}$ is the probability of being in the `good" aggregate state, given that the bank is able to pay $\bar{r}$. A straightforward application of Bayes rule yields:

$$W_G(\bar{r}) = \frac{\Pr ob\{r_1 \geq \bar{r}|G\}p_G}{\Pr ob\{r_1 \geq \bar{r}|G\}p_G + \Pr ob\{r_1 \geq \bar{r}|B\}(1-p_G)} \tag{20}$$

with $W_B(\bar{r}) = 1 - W_G(\bar{r})$ and $\Sigma_A = E(\sqrt{r_2/r^+}|A), A = G, B$

Σ_A is now the, normalized expected utility of the risky return in state A (also notice that, in comparison to $Q_{\bar{r}}, \Sigma_A$ only depends on the aggregate state).

3. Problem (P2) can be solved in two steps: first, the optimal c_{ij} 's and a are determined for $\bar{r}$ given, then the optimal $\bar{r}$ is selected by picking the promised return that yields maximal utility.

Notice that (P2) offers a large number of degrees of freedom and, with little loss of generality, the analysis will focus, until the end of section 4, on the case where $\alpha = 0$.[10]

Lemma 1 is useful to that respect. It shows that there will be no intermediation in equilibrium if the return on the risky asset is toc' low; in addition, $\alpha = 0$ is implied by a sufficiently high return on the risky asset.

Lemma 1 *The* solution of *program (P2), for* $\bar{r}$ *given, is such that (1)* If $0 \leq \alpha \leq 1$, $c_{11} \geq c_{21}$ *and* $c_{22} \geq c_{12}$. *(2a)* $\bar{r}$ *small implies* $\alpha = 1$; *(2b)* $\bar{r}$ *large implies* $\alpha = 0$; *(2c)* $\bar{r}$. *`intermediate' (to be defined in the proof) au.d* $P_{\bar{r}}$. *and* $Q_{\bar{r}}$ *large imply that 0* $< \alpha < 1$.

Proof:

2a: obvious. In that case, investing in deposits is always dominated by the storage technology, hence α=1.

2b-2c: see Appendix A.1 and A.2.

1: If $\alpha = 1$, $c_{11} = c_{12} = 1$ and $c_{21} = c_{22} = 0$. But for $0 \leq \alpha < 1$, the proportionality coefficient between c_{11} and c_{12} (defined as ε in Appendix A) is such that $\varepsilon < 1$. Hence $c_{11} > c_{12}$. Plugging this inequality in the first incentive constraint yields $c_{21} < c_{22}$.

The solution to problem (P2) is therefore given by plugging the optimal value of consumption in the utility function, and maximizing utility over $\bar{r}$.

Proposition 1 For Σ_G *and* Σ_B *sufficiently large so that* α *= 0, the optimal* $\bar{r}$ *in problem (P2) is given by* $\bar{r}^* = \arg\max_{\bar{r}} U(\bar{r})$, *where* $U(\bar{r}) = P_{\bar{r}}\theta_{\bar{r}} + U_0(\bar{r}, L_\upsilon)$ *is a hump shaped curve with* $\theta_{\bar{r}} = \Theta(Q_{\bar{r}}, \lambda_{\bar{r}}, \rho, p)$ *and* $\lambda_{\bar{r}}$ *is the Lagrange multiplier associated with the first incentive constraint in (P2), for* $\bar{r}, \rho$ *and p given.*

Proof: See Appendix B.3

3.3 Equilibrium under no communication

It is necessary to make it clear that the equilibrium described in Proposition 1 is not the only one allowed by the model. As in the DD model, given the Sequential Service Constraint, there is also a `run' equilibrium, where all depositors withdraw the type 1 allocation. This occurs if $L_\upsilon$ is small enough, so that the liquidation of the asset would not be sufficient to compensate all depositors equally in case of failure. This type of `run' based on a sunspot, has to be distinguished from the information-based run, introduced in section 4.

3.4 Numerical examples

1. Plots of $U(\bar{r})$, the expected utility of a representative depositor if the equilibrium of proposition 1 obtains, are exhibited in Figures 2 to 5. A truncated normal law is chosen for the

distribution of returns. $F_A(x)$ is defined as the cdf of $N(m_A, \sigma_A)$, $A = G, B$ and $m_G \geq m_B$. A discrete approximation is taken by defining a grid on $[r^-, r^+]$ with $r^- = 0$ and $r^+ = 1.5$. When $\bar{r}$ is increased parametrically on the range of r, U($\bar{r}$) is typically hump-shaped. The vertical dashed line selects $\bar{r}$, the value of r for which U(r) is maximized. When $p_G = .5$, $\bar{r}$ is usually located in the neighborhood of the mid point between m_G and m_B. When $p_G = .9$, $\bar{r}$ is closer to m_G.

2. If restrictions on *ex ante* risk-pooling are relaxed, it is possible to show that the welfare of depositors increases. There is no sucll systematic relationship for intermediaries. Depositors would favor more ex ante risk-sharing under the constraint that a collusive equilibrium does not become more likely (see section 5).

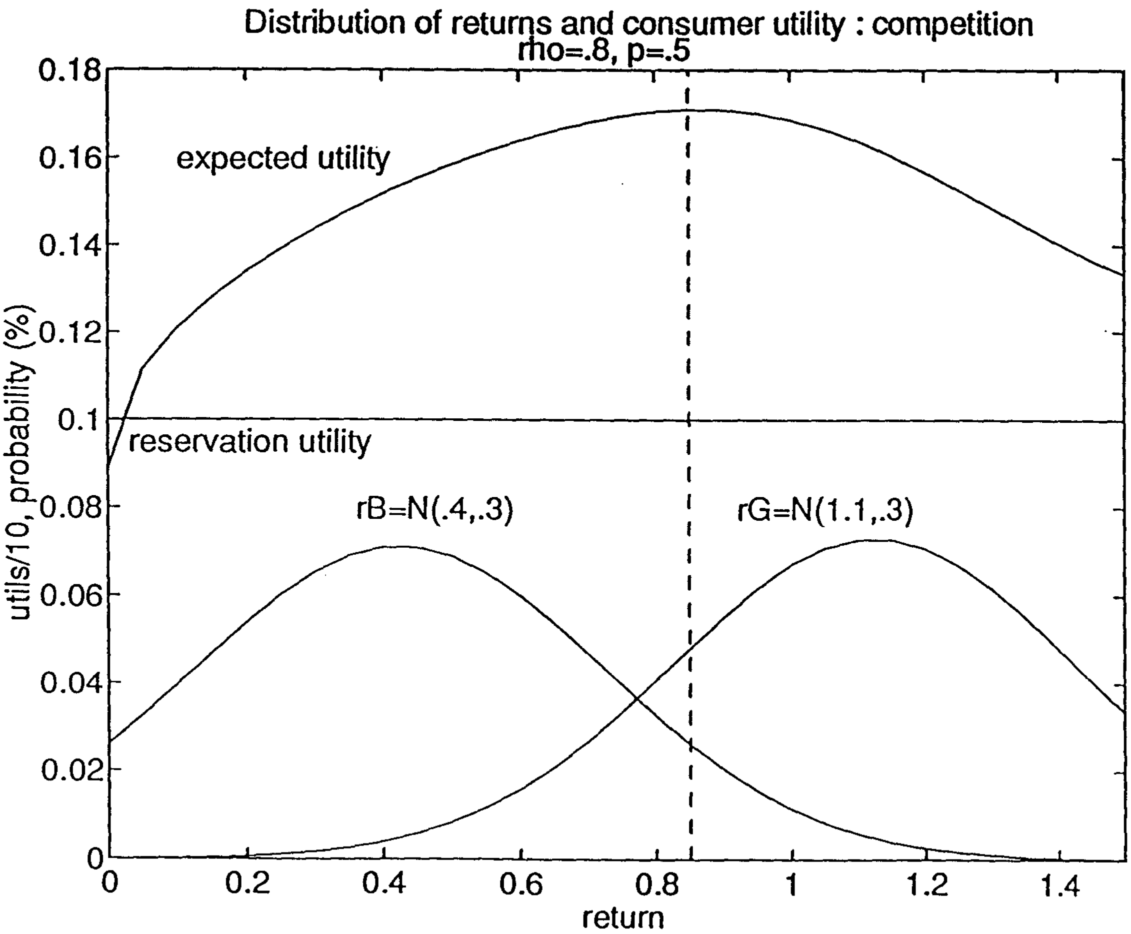

Figure 2: Optimal level of the promised return : numerical simulations ($p_G = .5$)

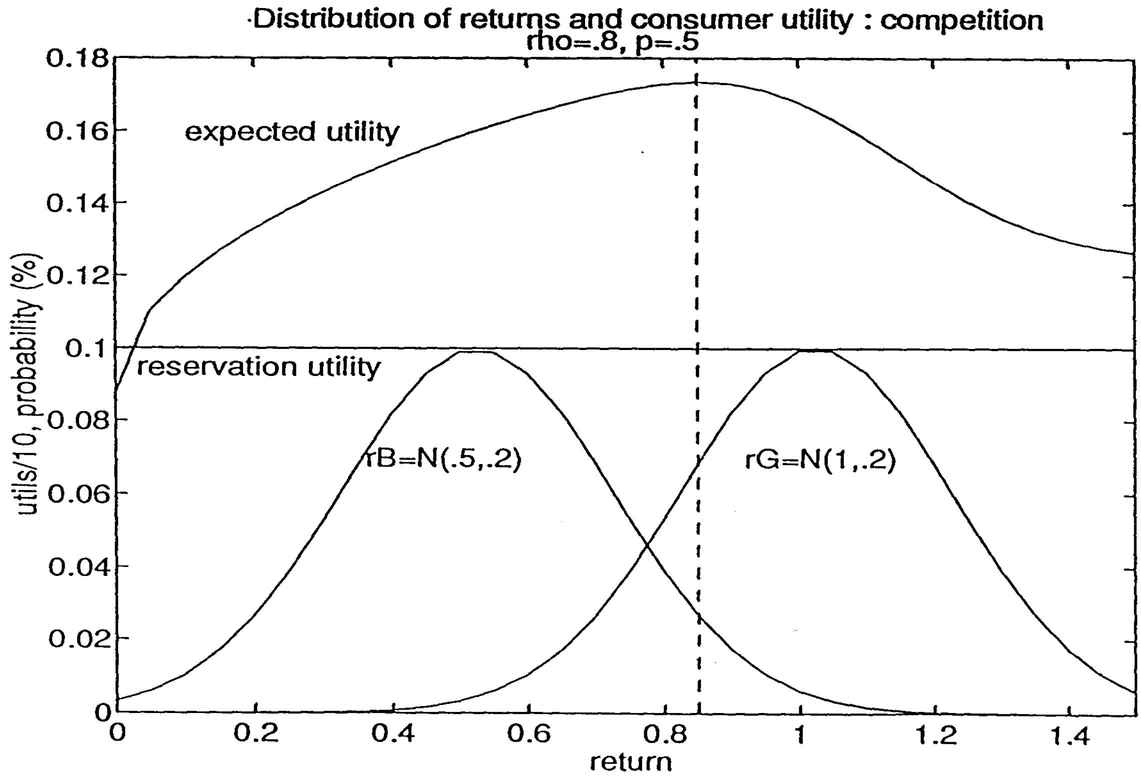

Figure 3: Optimal level of the promised return : numerical simulations (p_G = .5)

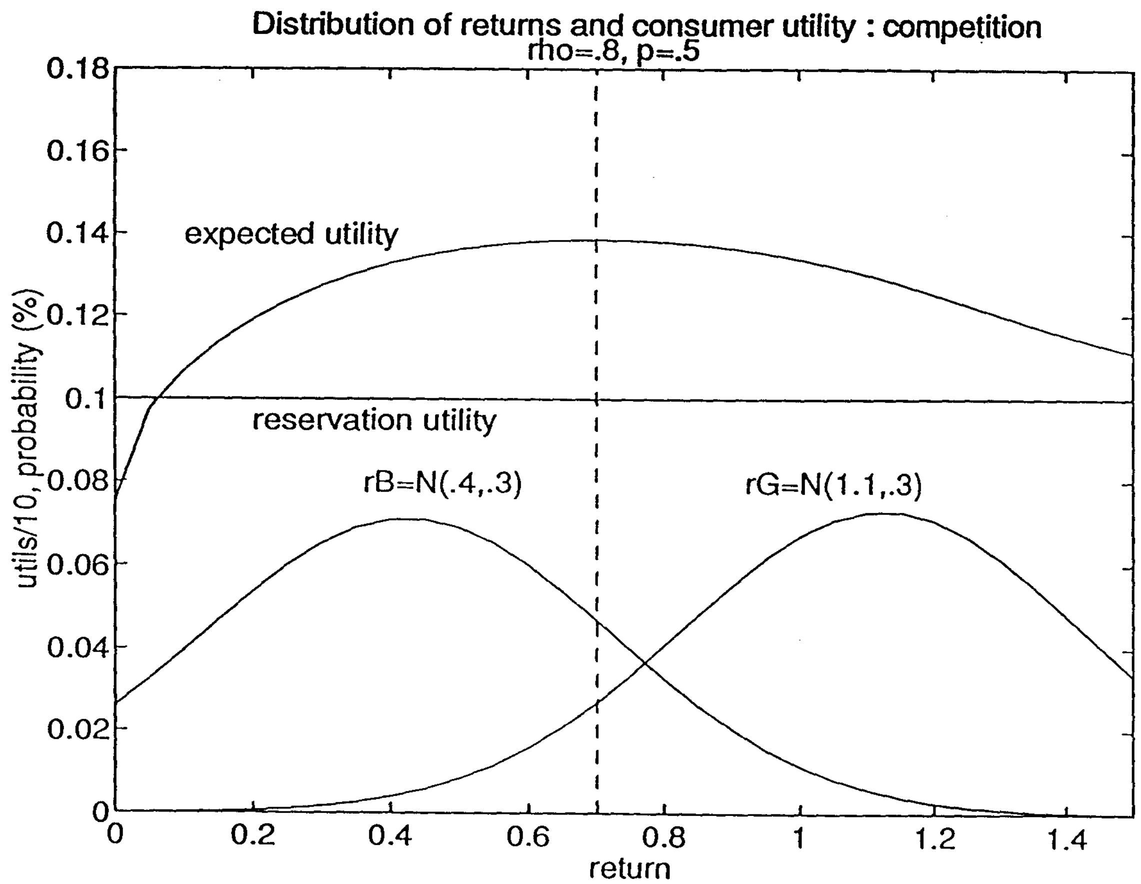

Figure 4: Optimal level of the promised return : numerical simulations (p_G = .9)

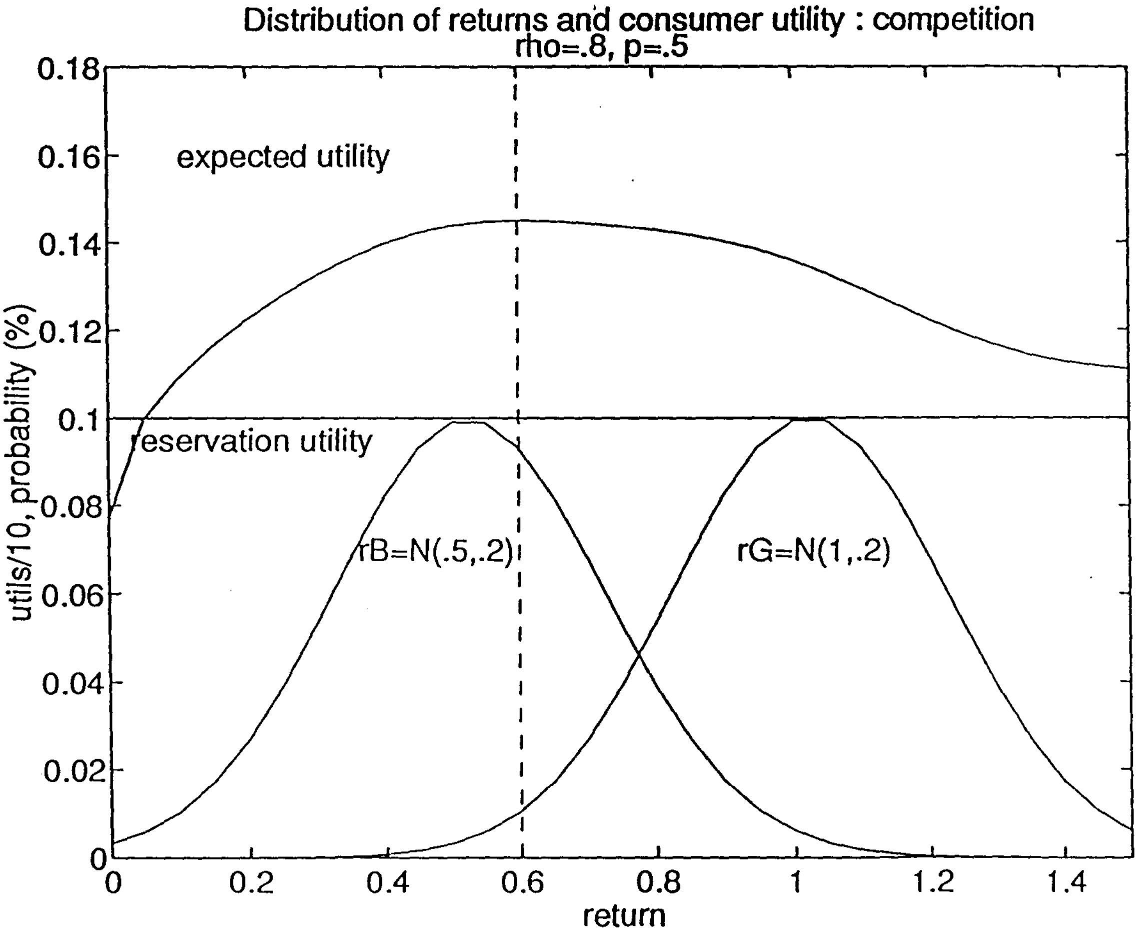

Figure 5: Optimal level of the promised return : numerical simulations (p_G = .9)

As a conclusion to this section, it appears that, when depositors have no information about other banks, the model, though richer than previous models (with multiple banks, a larger state space and a non trivial liquidation value), yields the same types of equilibria.

However the solutions are parameterized by the degree of ex ante risk pooling allowed by the model and there are always welfaxe gains for depositors from becter ex mate risk-pooling. In. that respect, the model is consistent, on the one hand, with observations that the major cause of the historical instability of U.S. banks was the unit-bank system (see Calomiris(1991), among others) and, on the other hand, with the current movement towards deregulation, in particular when some of the restrictions on risk pooling are relaxed (e.g. in the U.S. restrictions on interstate branching have been progressively lifted). Nevertheless, as argued in 2.2, it is reasonable to assume that there is a fraction of uninsurable risk. Therefore, the model of this section therefore provides a convenient benchmark to measure the eflect of (1) information-based runs and (2) ex post risk-sharing.

4 Case 2: Competition ex ante, communication ex post

Now the assumption of no-communication is relaxed. Depositors are informed at $t = 1$ about the ability of all banks to pay the promised return. However, it is assumed that the contract is not contingent on the availability of information [This simplifies the analysis, but the results would be robust to the alternative assumption, namely that the existence of new information at $t = 1$ is included in the contract at $t = 0$] and depositors cannot agree at $t = 1$ about a revision of the contract. Notice, however, that type 2's are more sensitive to the stochastic nature of the problem [See remarks in section 6 on that point.]

Before generalizing to K banks, let us statt with two banks k and 1, as in ABD, but the two banks are assumed to be totally symmetric: both banks have the same probability to fail. Without loss of generality, let us assume that 1 is unable to pay the promised return. From Proposition 1, this may occur in both cases of aggregate shock but, depending on the distribution of r, with a higher probability of occurence in state B than in state G.

When depositors in one bank get to know whether or not the other bank makes the promised payment at. $t = 1$. k's depositors will update their belief about the proba.bility of failure of their bank at $t = 2$. Depositors may decide to withdraw at. $t = 1$ more than what was agreed upon ex ante and `run' their bank, if they believe that $r_2$ is likely to be small. The availability of information about the performance of other banks gives a precise content to the concept of `intermediary information' in the JB model. Friedman and Schwarz(1963) provide substantial evidence of how distrust of banks degenerated in banking panics during the National Banking System and the Great Depression. Similarly F. Mishkin(1991) concludes his review of the causes of financial crisis in the U.S. by noticing that *"the onset of many panics followed a major failure of a financial institution"* [Mishkin(1991), p. 96.], even if it was not necessarily a bank. But, consistently with my assumption that the cause of a failures was a bad returns on a. "project", lie also adds that *"this failure was often the result of financial difficulties experienced by a non financial corporation"* [Ibid., p. 96.].

In the model, runs need not lead to bankruptcy. The latter is only declared in the case where the intermediary is not able to meet the demand for withdrawal. This feature distinguishes my model from JB, where the intermediary pays to the depositors the whole return on the portfolio. Here, even if it is not optimal *ex ante,* the intermediary may use *ex* post, in certain states of nature, a fraction of its net return to repay any supplementary demand for withdrawal.

The objective of this section is, therefore, to give conditions under which (1) new information will trigger runs, and (2) intermediaries will effectively be sensitive to runs, i.e. fail when a run occurs. In that context, De Bandt(1994, Chapter 5) studies whether alternative strategies to counter runs are feasible.

4.1 Belief-updating and run on intermediaries

I proceed in two steps. First I extend the JB model to a larger state-space by determining how the update of beliefs about the probability of occurence of the two aggregate states may trigger a run. Then I apply this result to determine how many banks have to rail to trigger a run [I focus on failing banks, since it may induce type 2 depositors to prefer the type 1 allocation. The case where type 1 would prefer the type 2 allocation, if it appears that the state *G* is very likely, does not matter to the bank, whicli is always able t.o pay the type 2 allocation to both types when $r \geq \bar{r}$.].

4.1.1 Intermediate information

When type 2 depositors -who receive intermediary information Jl- update their beliefs about the probability of being in state *G*, incentive constraint (17) in (P2) may be binding or even reversed, if type 1 allocation become the mort. preferred one. Let $Q'_{\bar{r}}$ be the revised value of $Q_{\bar{r}}$, and:

$$Q'_{\bar{r}} = W'_G(\bar{r})\Sigma_G + (1 - W'_G)(\bar{r})\Sigma_B \qquad (21)$$

with $W'_G(\bar{r})$ the revised probability of being in state *G* given J_1. Proposition 2 gives conditions on $W'_G(\bar{r})$ under which one can expect a run. This is a straightforward extension of the JB result to my setup.

Proposition *2 If $W'_G(\bar{r})$ is the revised probability of being in state G given intermediate information. J_1, a run occurs on all intermediaries iff*: $w'_g(\bar{r}) < \rho W_G(\bar{r}) - \frac{1-\rho}{\omega}$ *where* $\omega = \frac{\Sigma_G}{\Sigma_B} - 1 \geq 0$

Proof:

A run occurs when type-2's prefer type 1 allocation and withdraw c_{11} at $t = 1$, instead of c_{12}. If type-2 consumera prefer type-1 allocation, incentive constraint (6) of (P2) is reversed. It lniist be true that:

$$U(c_{12}) + U(c_{22})Q'_{\bar{r}} \leq U(c_{11}) + U(c_{21})Q'_{\bar{r}}$$

Rearranging ternis yields: $Q'_{\bar{r}} < \frac{U(c_{11}) - U(c_{12})}{U(c_{22}) - U(c_{21})} = \rho Q'_{\bar{r}}$ where the last equality follows from constraint (16). Developing the first and last terni of the preceding inequality, using (19) and (21), one gets:

$$\frac{\Sigma_G}{\Sigma_B} > 1 + \frac{1-\rho}{\rho W_G - W'_G}$$

and ω ≥ 0, since $F_G(r)$ stochastically dominates $F_B(r)$.

When ρ is not too different from 1 (type 1 also smoothes consumption over time), and ω is large (i.e. the two distributions are very different), W'_G need not be significantly smaller than W_G to trigger a run.

It is necessary to make it clear that this result resta on the absence of coordination among depositors. As we shall see later on, mis may drive an intermediary to failure. In the model, each depositor has a zero-measure and her individual beliavior has no effect on the intermediary's equilibrium. Consequently a run is always a Nash equilibrium.

A simple proof of this result goes as follows. Let us assume that the conditions of proposition 2 are satisfied. On t.he one Rand, running is always optimal for an individual depositor when otlier depositors decide not to run, rince failure does not occur. On the other lland, if the other depositors decide to run, given the S.S.C. it is optimal for an individual depositor to run too, in order to get something at t = 1, if $L_\upsilon \approx 0$ .If L_υ is large, she is indifferent between running and not running: her behavior does not prevent. the bank from failing. Consequently, in comparison to the `run' equilibriuin of section 3, information-based runs do not depend on $L_\upsilon$. It remains that, although 'no run' is Pareto superior to `run', this equilibrium will not be sustainable under the pressure of depositors' individual behavior, once the conditions of Proposition 2 are met.

4.1.2 Threshold nuniber of failing intermediaries

In the general case, W_G' will meet proposition 2's conditions if more than L^* of the K intermediaries are failing. From the point of view of a depositor in a bank which does not suspend, say bank k, and observing the K - 1 ot lier banks, J_1 is now the event F_{K-1}^L : "L banks among the K - 1 are failing and the Kth bank is not. failing". Let. us write

$$W_G' = W_G(K-1, L, \bar{r}) = prob\{G | F_{K-1}^L\}$$

Using Bayes rule:

$$W_G(K-1, L, \bar{r}) = \frac{prob\{F_{K-1}^L | G\} p_G}{prob\{F_{K-1}^L | G\} p_G + prob\{F_{K-1}^L | B\}(1 - p_G)} \tag{22}$$

and

$$prob\{F_{K-1}^L | G\} = (1 - F_G(\bar{r}))\left[\binom{K-1}{L}(F_G(\bar{r}))^L (1 - F_G(\bar{r}))^{K-1-L}\right] \tag{23}$$

where the second part of the preceding formula is simply given by a binomial distribution.

A run will occur if L $\geq L^*$, where L^* *is* the smallest L such that $W_G(K-1, L, \bar{r})$ meets the condition stated in Proposition 2.

4.2 Sensitivity to runs

We know from Lemma 1, that $c_{11} > c_{12}$. Let us focus, as before, on the case where $\alpha = 0$. In case of a run, bank k will fail if withdrawals exceed the amount of cash available to the bank. Without cooperation among banks to provide resources, the primary source of cash is made of the proceeds at t = 1 from the investrnent made at $t = 0$.

Proposition *3 Under perfect competition and without cooperation among banks, bank k will fail after at the end of t = 1, when $\alpha = 0$, if*

$$\phi(\bar{r}^*)\bar{r}^* > r_1^K$$

where $\phi(\bar{r}^*) = (p + (1-p) \in (\lambda_{\bar{r}^*})^2)^{-1}; \lambda_{\bar{r}^*}$ *is the Lagrange multiplier associated with the first incentive constraint in (P2), for an optimal promised return of* $\bar{r}^*$*; and* r_1^K *is the return, at t = 1, on k's investment.*

Proof: Bank k will not. be able to counter a run if

$$c_{11} > r_1^K$$

Plugging into the resources constraint (14) in (P2) the expression of c_{12} as a function of c_{11} derived from the FOC's, namely $c_{12} = \varepsilon(\lambda_{\bar{r}^*})c_{11}$, one gets the desired result.

To summarize, the return on the risky asset may fall in one of the three regions exhibited in figure 6:

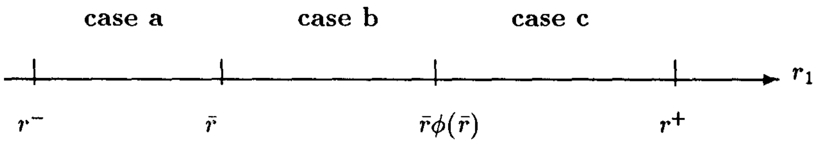

Figure 6: Sensitivity to a `run' given the return on the risky asset at t = 1.

- Case a.: r < $\bar{r}$. The bank is insolvent and suspends (`fundamental failure').
- Case b: $\bar{r} \le r < \bar{r}\phi(\bar{r})$. A `run' implies failure.
- Case c: $r \ge \bar{r}\phi(\bar{r})$. A `run' does not imply failure.

[These three conditions would be similar in the case wllere 0 < α < 1: a bank would fail from a run if $r < \frac{\alpha}{1-\alpha}(\phi(\bar{r}) - 1) + \phi(\bar{r})\bar{r}$, where the first term of the RHS is positive since $\phi(\bar{r}) \ge 1$.]

Two additional remarks need to be made about the existence of runs. In the first place, as indicated in section 2, each individual bank has private information on the realization of the return on its risky `project', but we have assumed in section 3 that substantiel information is released about returns through the revelation by banks in different locations of their ability to pay the promised return $\bar{r}$. It would be unrealistic to assume that depositors have superior information and that banks do not know the announcements made by other banks. However, the assumption that each individuel bank also gets to know the number of `fundamental failures' has an incidence on the existence of runs. On the one hand, each bank can perfectly anticipate the behavior of depositors, i.e. whether or not they will run. If banks know that depositors are going to run w.p. 1, they will try to suspend immediately, before withdrawals cake place. Therefore mils -and not purely contagious failures- will appear in two cases: (1) if banks do not observe what other banks do when they make their announcement about their solvency; and (2) if depositors do not have perfect knowledge of the number of failing banks (e.g. they only get. t.o know the value of L with probability q, with $q \approx 1$). In the latter case, b banks do not want to lose their expected profit of $(1 - q)(r^k - \bar{r})$ (and to pay any possible failure cost) so that they prefer to face a run [See section 2 for further details.].

In the second place, as noted in section 3, $\frac{d\lambda}{d\bar{r}} \leq 0$ and $\frac{d\epsilon(\lambda)}{d\lambda} \leq 0$. Therefore when $\bar{r}$ increases, $\varepsilon(\lambda)$ increases slowly, hence $\phi(\bar{r})$ is slightly reduced. Therefore, as $\bar{r}$ is increased, $\phi(\bar{r})$ becomes smaller. Thus the range of values of r over which a run may threaten an intermediary is smaller for a higher level of $\bar{r}$. This compensates partially the fact that the inability to pay the promised return, namely $prob\{r < \bar{r}\}$, is increasing in $\bar{r}$. Also notice that $\phi(\bar{r})$ is decreasing in ρ, since the two types of consumption are much more similar when ρ is closer to 1. Consequently $\phi(\bar{r})$ measures the risk of run associated either with a low level of the parameter ρ, or with a low equilibrium value of $\bar{r}$.

Tables 1 to 3 provide numerical simulations of the value of $\phi(\bar{r})$., In Table 1 and 2, $\phi(\bar{r})$ is smaller for ρ = .85 than for ρ = .8. Table 3 only differs with respect to p_G.

Parameter values			Equilibrium values		
ρ	m_B (σ_B)	m_G (σ_G)	$\bar{r}$ ($\phi(\bar{r})$)	W'_G (Σ_G/Σ_B)	L^* ($K = 21$)
.8	0.50 (0.20)	1.00 (0.20)	0.60 (1.1141)	0.0874 (1.4158)	5
.8	0.50 (0.30)	1.00 (0.30)	0.65 (1.1105)	0.0507 (1.3908)	9
.85	0.50 (0.20)	1.00 (0.20)	0.60 (1.0848)	0.2432 (1.4158)	6
.85	0.50 (0.30)	1.00 (0.30)	0.65 (1.0823)	0.2138 (1.3908)	9

Table 1: Failure threshold :numerical simulations(p_G=.5,p=.5)

Parameter values			Equilibrium values		
ρ	m_B (σ_B)	m_G (σ_G)	$\bar{r}$ ($\phi(\bar{r})$)	W'_G (Σ_G/Σ_B)	L^* ($K = 21$)
.8	0.40 (0.20)	1.10 (0.20)	0.90 (1.1095)	0.4833 (1.6541)	16
.8	0.40 (0.45)	1.10 (0.45)	0.65 (1.1091)	0.0635 (1.4120)	10
.85	0.40 (0.20)	1.10 (0.20)	0.90 (1.0815)	0.6090 (1.6541)	15
.85	0.40 (0.45)	1.10 (0.45)	0.65 (1.0813)	0.2192 (1.4120)	10

Table 2: Failure threshold :numerical simulations(p_G=.5,p=.5) (continued)

Parameter values			Equilibrium values		
ρ	m_B (σ_B)	m_G (σ_G)	$\bar{r}$ ($\phi(\bar{r})$)	W'_G (Σ_G/Σ_B)	L^* ($K = 21$)
.8	0.50 (0.45)	1.00 (0.45)	0.75 (1.1078)	0.0491 (1.2822)	15
.8	0.40 (0.45)	1.10 (0.45)	0.80 (1.1080)	0.2852 (1.4120)	14
.8	0.50 (0.30)	1.00 (0.30)	0.80 (1.1090)	0.2653 (1.3908)	13
.8	0.40 (0.30)	1.10 (0.30)	0.85 (1.1098)	0.4493 (1.5876)	14

Table 3: Failure threshold :numerical simulations(p_G=.9,p=.5)

4.3 Suspension of convertibility

As indicated in the preceding subsection, runs cause the failure of *b* banks in addition to the `fundamental failures' of case a. Since liquidation implies the interruption of the risky project it is interesting to see whether, in addition to being incentive compatible on an individual basis, such an outcome is also Pareto optimal.

The question is not relevant for *c* banks which do not fail. Given the information available a run does not cause failure. Hence, it is socially optimal to run given the information available. When banks cannot credibly reveal the aggregate state (banks would always declare that the state is `good'), runs may even take place in the `good' state, but they are nevertheless rational, in the sense that they are not based on a sunspot.

For *b* banks, the problem is different. Let us determine the conditions under which a social planner prefers liquidation to continuation of *b* banks, when the aggregate state is `bad'.

When L_υ is large enough to compensate all depositors equally in case of liquidation (i.e. when $L_\upsilon \geq \phi(\bar{r})\bar{r} - r$, with r $\geq \bar{r}$), we need to compare $U(r+ L_\upsilon)$ and $U(c_{12}) + \rho Q'_{\bar{r}} U(c_{22})$, for $r \geq \bar{r}$.

We know from Lemma 1 that $\bar{r} > c_{12}$, hence $U(r) > U(c_{12})$. In addition, equation (9) is equivalent to U(L_υ) $\leq \rho Q_{\bar{r}} U(r^+)$, with $c_{22} > r^+$. It implies that U(L_υ) $\leq \rho Q_{\bar{r}} U(c_{22})$. However, due to the concavity of the utility function, even for $Q'_{\bar{r}} \leq \rho Q_{\bar{r}}$ we generally have:

$$U(r + L_\upsilon) \leq U(c_{12}) + \rho Q'_{\bar{r}} U(c_{22}) \quad (24)$$

Nevertheless, in the case where depositors have a low degree of relative risk aversion (or a high degree of intertemporal substitution of consumption), equation (24) may be reversed if L_υ is such that

$$\rho \Sigma_B r^+ \leq L_\upsilon \leq \rho \Sigma_G r^+ \quad (25)$$

where Σ_B and Σ_G are defined in equation (19) and (20)): if depositors know that the aggregate state is `bad' w.p. 1 and L_υ is large enough, type 2 depositors always prefer liquidation to continuation of the risky project. Notice that the second inequality in (25) is implied by equation (9) for *U (C) = C*.

When L_υ is small $(L_\upsilon \geq \phi(\bar{r})\bar{r} - r)$, only the first depositors in line will receive c_{11}, at t = 1 and nothing will be handed out at t = 2. In that case, equation (24) is even more likely to hold.

In this framework, one type of coordination device would be to suspend convertibility: banks would allow depositors to withdraw type 1 allocation in proportion *p* only, and type 2 allocation beyond that point. Consequently, if batiks are unable to credibly reveal the aggregate state, suspension of convertibility is Pareto optimal for *b* banks' customers with L_υ small and depositors sufficiently risk-averse. Notice that in this framework all *b* banks suspend convertibility simultaneously, runs still occur, but do not lead to additional failures (some depositors do not receive the allocation they would like). This is consistent with many episodes of suspension of convertibility where banks were offered more room to, manoeuver, but the banking system was still held under suspicion so that runs were still observed.

If banks could credibly reveal the aggregate state, as in Gorton (19.85) -where information is transmitted to depositors at a cost-, suspension of convertibility would be sustainable in the `good' state. However, if this were true, the availability of. this information should be integrated in the initial contract and prograins (Pl) and (P2) should be changed accordingly.

To conclude this sub-section, the introduction of a policy of suspension of convertibility, in a model where banks cannot credibly reveal the aggregate state, yields a Pareto superior equilibrium, unless the liquidation value is very high or individual have a low risk aversion. This provides a partial explanation for the relative inefïiciency of suspension of convertibility in the prevention of runs in the U.S. during the Great Depression: during a protracted recension, the liquidation value of projects was so low that the interruption of projects was very costly, and no depositor opposed additional constraints on withdrawals. However on an individual basis, `bad news' induced depositors to try to avoid any further loss by trying to withdraw their own deposits.

The comparison between section 3 and 4 reveals that more information (the observation of `fundamental' failures in the whole economy) may prove detrimental to social welfare. As mentionned above, this results from a framework where markets are incomplete (liquidation of the risky asset at t = 1 is costly) and, without coordination, depositors use the available information non cooperatively.

5 Dynamics

Two problems remain unresolved in the preceding section. First, the strong result about Bertrand competition may not hold any more if repeated interactions are allowed. Secondly, in a one-shot setting, it is difficult to mobilize anything else than the excess return of the intermediaries which would sufter from a run. Actually there is evidence of inter-period risk sharing among all banks.

The basic model of the preceding sections is therefore extended to a multi-period setting. There is an infinite succession of 3-period investments. At $t = 0, 3, \ldots 3n, \ldots,$ consumers are endowed with one unit of the consumption good and investment choices are made. At $t = 3n+1$ a promised return is paid and the project is liquidated at $t = 3n + 2,$ for $n = 0,1,2,\ldots.$ It is assumed that intermediaries collect deposits at $t = 0, 3, \ldots. 3n, \ldots,$ if they do not fail in the preceding period." Furthermore, intermediaries discount intertemporal utility at the rate ,(β < 1 per period.

In this repeated game setting, two types of arrangements among intermediaries may be possible: monopoly and interbank risk-sharing.

The section is organized as follows: using the equilibrium concept of subgame perfection, conditions are presented under which each of these two types of arrangements, taken one by one, are self-sustainable. Their joint sustainability is examined in conclusion.

5.1 Collusion on the level of the promised return

If the number of intermediaries is fixed and entry limited to the replacement of failing intermediaries, with repeated interactions among intermediaries the monopoly level of promised return $\bar{r}^M$ at $t = 1$ will be enforced.[12]

Depositors will only receive their reservation utility and the problem to solve now is to maximize intermediary k's expected excess return under this constraint. Program *(P3)* is therefore the following:

$$\max_{c_{i,j},\alpha,\bar{r}^M} (1-\alpha)E(r_1^K - \bar{r}^M \left| r_1^K \geq \bar{r}^M \right.) prob\left\{ r_1^K \geq \bar{r}^M \right\} \tag{26}$$

$$\text{s.t.} \qquad u(\bar{r}^M) \geq 1 \tag{27}$$

and constraints (12), (14), (15), (16) of (P2) hold with $\bar{r}^M$ substituted to $\bar{r}$.

In (P3), like in (P2), the deposit contract provides insurance against being a type 1. But the objective function of (P2) becomes a constraint in (P3).

We know that the choice α = 1 is a trivial way to provide depositors their reservation utility, but in Chat case the representative intermediary makes no profit. Therefore, this cannot be a solution to (P3), unless the return on the risky asset is so low that it is always dominated by the safe asset (there is no intermediation). Conversely, in the case where α = 0, the intermediary's utility is the total expected excess return $E(r - \bar{r}^*),$ where $\bar{r}^*$ is the minimum level of the promised return at $t = 1$ such that the representative depositor's utility is equal to 1. Figures 1 to 4, associated with problem (P2) -in the case α = 0- provide evidence that, depending on the distribution of r, $\bar{r}^*$ may be small so that the risk of failure $prob\{r < \bar{r}^*\}$ is close to zero. However, this is not yet the optimal solution.since, according to the following lemma, the intermediary gets higher utility by choosing $\bar{r}^* \approx 0$ and $0 < \alpha < 1$. This is the result of a comparison of the intermediary's expected excess return in the two cases $\alpha = 0$ and $0 < \alpha < 1$, for different values of $\bar{r}$. The intuition is that depositors are risk averse, so that an increase in F means more risk. The loss for intermediaries in terms of expected excess return $E(r - \bar{r})$ is larger than the increase in depositors utility.

Lemma 2 *With a square-root utility function for depositors, monopoly power by intermediary implies that* $\bar{r}^M \approx 0$, *hence the risk of failure is almost null.*

Proof: See Appendix C.

Static gains from collusion are measured by comparing expected net return in the monopoly case and the perfect competition case. Long run gains will depend on the capacity to remain in the market many periods.

Proposition *4 When the number of intermediaries is fixed, with K not too large, collusion on the level of the promised return is a subgame perfect equilibrium if the probability of failure is small and* β *is sufficiently large.*

Proof (intuition): One-period gains from collusion (ΔG) are always positive. Therefore multiperiod collusion is sustainable if the gains from deviating at $t = 3n$ are smaller Chan the loss from the non-cooperative equilibrium. Deviation allows to increase market share from 1 to K.

Let $E = E(r_1 - \bar{r} | r_1 \geq \bar{r})$ and $E^M = E(r_1 - \bar{r}^M | r_1 \geq \bar{r})$, be the expected excess return in competition and monopoly, respectively.

$\pi = P_{\bar{r}^M}$ and $\pi^M = P_{\bar{r}^M}$ are the corresponding probability of being able to pay the promised return. Of course $\pi \leq \pi^M$ and $E \leq E^M$.

At $t = 0$, collusion is a subgame perfect equilibrium if:

$$\beta K \pi E \leq \sum_{j=0}^{\infty} \beta^{3j+1} (\pi^M)^{j+1} E^M$$

Or, equivalently, $K\pi E \leq (\pi^M E^M)/(1 - \beta^3 \pi^M)$. π^M is likely to be close to one under monopoly, as indicated in Lemma 2. However collusion will not be sustainable for K too large, unless substantial costs are incurred by depositors switching their account from one bank to another.

The size of K is an important factor explaining differences between U.S. versus Canadian and European banking systems. Putting restrictions to concentration is one way to prevent monopoly power, but it is not the only variable that lias to be taken into consideration to achieve this goal: the distribution of r (and its effect on failure rates) and the level of β play a crucial role, too.

5.2 Interbank risk-sharing

The point about the effect of intermediaries' long term view can also be made with the introduction of risk-pooling. The analysis focuses therefore on the case where some intermediaries, not threatened by a potential run in the current period, provide cash to other intermediaries facing adverse idiosyncratic shocks at $t = 1$.

Borrowing and lending should be possible, but to simplify, 1 focus on pure risk-sharing through reserve-pooling: reserves are provided by intermediary k with positive excess return at $t = 1$ if the other intermediaries can credibly commit to give assistance at a later date in the case where k would need it.[13]

The same argument as for the level of the promised return (section 5.1) can be made here. Two effects have to be distinguished. First the direct effect of a bail out: banks stay in the market and do not lose the expected future return. Second there is an indirect benefit that enjoy banks with $\bar{r} < r < \bar{r}\phi(\bar{r})$, as described in section 4.

As it appears in Proposition 5, a major impediment to the d.evelopment of interbank cooperation is the existence of too, large a risk of failure which prevents banks from having long term objectives.

Proposition *5 For β sufficiently large, ex post interbank risk-sharing through reserve-pooling is a subgame perfect equilibrium if the probabilily of failure is very low.*

Proof (intuition): Let us assume that banks agree to provide a maximum of C to bail out other failing baillis. Let us define $E_1 = E(r_1 - \bar{r} | r_1 \geq \bar{r})$ and $E_2 = E_1 - \tilde{p}_1 C + \tilde{p}_1(C + C_{\upsilon 1}) + \tilde{p}_2 C_{\upsilon 2}$, where $\tilde{p}_1$ is the probability of observing assistance to failing banks and $\tilde{p}_2$ the probability that thanks to the assistance secured by failing banks, banks with $\bar{r} \leq r < \phi(\bar{r})\bar{r}$ are able to avoid a run. The last two terms in E_2 measure the `direct' and the `indirect' effect of a bail out (failure cost F_c, is assumed to be negligeable).

Let π_2 *and* π_1 be the probability of no failure with and without risk-sharing arrangements: $\pi_2 = \pi_1 + \tilde{p}_1 + \tilde{p}_2$. The discounted sum of future expected returns is given by: $C_{\upsilon 1} = \sum_{j=0}^{\infty} \beta^{3j} (\pi_1)^j E_1 = \frac{E_1}{1-\beta^3 \pi_1}$ without risk -sharing and $C_{\upsilon 2} = \sum_{j=0}^{\infty} \beta^{3j} (\pi_2)^j E_2 = \frac{E_2}{1-\beta^3 \pi_2}$ with risk sharing.

Risk-sharing is a subgame perfect equilibrium if $C + C_{\upsilon 1} \leq C_{\upsilon 2}$ This condition will only hold if π_2 and β are close to one, and C is small.

The preceding subsections have shown that the two types of arrangements are self-sustainable, under conditions on β and K and the probability of failure.

Cooperative risk-sharing including intermediaries that are not likely to suffer from a run requires a very low failure rate. There are two ways to meet this condition:

- Men the monopoly. price is enforced, as indicated in lemma 2, the failure rate is almost nul].

• When the LLR bails out failing banks if intermediaries are not endowed with enough resources to bail out a significant number of them.

In botte cases, it is necessary that such the amount of risk-sharing be bounded -that the commitment to bail out be limited- for the intervention to be incentive compatible. The more prevalence of interbank risk-sharing in monopolistic banking system (e.g. in Canada, as opposed to the U.S..) -is consistent with those observations.

6 Discussion and conclusion

6.1 Results of the paper

1 An attractive feature of the model is its ability to distinguish between `fundamental' bank failures.

-when the return on the risky asset is lower than the promised return at $t = 1$- and `runs'

-where `healthy' intermediaries fail if depositors update their beliefs about the aggregate state, pence the expected return at $t = 2$. In addition, a `run' on a bank need not lead to bankruptcy, if the return on the risky asset is high enough to meet the demand for withdrawal. According to historical accounts of banking panics, some banks were able to weather runs and remain open.

2. The model highlights the role of externalities in the intermediation process. It is therefore consistent with attempts to organize mutual assistance among banks to limit contagious failures. Historical accounts show Chat banks used to act cooperatively when a banking panking panic was imminent. In addition, in order to build up expectations that are favorable to the stability of the banking system, as little publicity as possible was made about rescued banks. As indicated by Cannon (1910) *"attempts from the business community were made in vain to discover what banks had taken out in (loan) certificates"*.[14]

The model also rationalizes some features of suspension of convertibility, which can partially alleviates the effects of the lack of coordination among depositors.

3. The model shows that two configurations are possible for the organization of the banking industry. First, if there is a high level of competition in the banking industry, banks have a low level of reserves that could be used in case of financial panics. In addition, their horizon is short given the high risk of failure. In that case, banks expect very little from a bailout organized by the industry. ,.:Cooperative arrangements like clearing houses in the U.S. in the late 19th century and the early 20th century provided welfare gains, but they were small. This can. explain subsequent reforms and the search for other solutions than purely "professionnal" to the recurrent instability in the banking system. Second, when there is some degree of collusion among banks, as in Canada or in many European countries, cooperative arrangements to limit the risk of failure are more likely to emerge as an optimal organization of the industry.

6.2 Robustness of the model

The following remarks can be made:

First of all, the model assumes that investment choices take place at $t = 0$, where a one-period safe asset and a two-period risky asset are available. In the model, this implies that the allocation of type 2 agents (who value consumption at $t = 2$ most) is more stochastic than the one of type 1. Type 2's are therefore more sensitive to `news' about the aggregate state. This property still holds if the safe asset is also available between $t = 1$ and $t = 2$. In that case, depositors' reservation utility increases, since depositors are able to smooth consumption over the two periods. Therefore, as indicated in lemma 1, the existence of intermediation in equilibrium will occur for a slightly smaller range of the parameters describing the risky technology. What is more important is that, since reinvestment decisions are made at the end of $t = 1$, intermediaries are now able to use a fraction of the excess return at $t = 1$, namely $r_1^K - \bar{r}$, to relax the resource constraint at $t = 2$.

This new problem can be mapped into (P2) by expanding the state space of the endowment at $t = 2$ (the LHS of equation (15)). Nevertheless, since each interrnediary has private information on the realization of his r_1^K, it would be difficult to monitor the reinvestment of $r_1^K - \bar{r}$.

2. Next, it might be desirable to introduce more risk sharing between depositors and intermediaries. In order to reduce the welfare loss associated with two many failures, $\bar{r}$ might be partially dependent on idiosyncratic shocks. Nevertheless, it is important that depositors do not observe the aggregate state. Again, allowing more dependence on the true realization of the returns would not affect the main results of the paper: `runs' would only be more frequent. However, it is consistent with actual banking practices to assume that at least a fraction of banks' liabilities are non contingent.

6.3 Further research

1. Further research includes the endogeneisation of the timing assumed at $t = 1$. The distinction, made in the model, between, first, the announcement by banks whether or not that they can pay r and, secondly, the withdrawals by depositors, is designed to avoid further assumptions about the flow of information received by depositors. In fact, even without explicit announcement, depositors are rapidly informed about the ability of banks to meet their contractual obligations: banks with low returns face a long line of depositors and, before withdrawing, some of the depositors can observe the number of banks that are experiencing adverse conditions in the whole economy. It remains that banks with returns close to, but less than $\bar{r}$ are likely to behave like those with r = $\bar{r}$. Consequently depositors may receive more information about the aggregate state. More `runs' may therefore occur than in the original model, unless banks try to avoid costly runs.

Another direction for future research is the study of moral hazard in the selection of projects, by introducing the choice by the manager of a level of effort, or equivalently between projects of different riskiness. This would endogeneise the level of ex ante risk, which is simply assumed in the model. It is clear that with multiple projects, perfect risk-sharing among banks would give the manager of an individual bank an incentive to choose the higher risk project. Incentive compatible risk-sharing would therefore imply a borrowing constraint on the amount of assistance a bank can require.

3. In addition, the extension of the model of section 5 to an overlapping generation framework should reveal interesting dynamics if intermediaries are allowed to use new deposits by `young' agents to repay old depositors.

FOOTNOTES

1: A more general CRRA utility function $\frac{c_{ij}^{1-a}}{1-a}$ with 0 < a < 1 may also, be chosen. a is, at the same time, the coefficient of relative risk aversion and the inverse of the elasticity of substitution between consumption at any two points in time.

2: A more explicit formulation of this assumption is the following: in case of failure of the project at t = 1, the banker incurs a non pecuniary desutility of F_c (bad reputation, loss in human capital, cost of a run). The only way to reduce failure costs is to declare insolvency to depositors at t = 1, i.e. before they are allowed to withdraw. In that case the banker saves his effort and failure costs are reduced to F_c - z, with z arbitrarily small. The last assumption is special but not essential. It is only designed to provide a clear signal to depositors, since banks with $r < \bar{r}$ know that they are going to fail w.p.l. Declaring insolvency is therefore optimal. Alternatively, if a run may occur with probability $q < 1$, it is not optimal to suspend early if $q(-F_c) + (1 - q)(r - \bar{r}) \geq -(F_c - z)$. When $r = 0$, *I* need that $(1 - q)F_c \geq z$, i.e. the expected loss from non-interruption is bigger than the gain from an early interruption. The setup could be extended by allowing revelation of insolvency when $r < r^* \langle\langle \bar{r}$, at the cost of straightforward but tedious algebra. The basic intuition would not be affected.

3: See Conclusion for the robustness of the model to this assumption.

4: It may be difficult to assume that banks finance their activity using deposits only. Since we have assumed competition among banks and perfect information at t = 2 about the returns, banks' profits should be equal to zero at t = 2, hence r_2^K is shared among depositors.

5: This realistic description of closing procedures reduces the information available to depositors when they witlidraw. In that case depositors only know that $r^k \geq \bar{r}$, or that $r^k < \bar{r}$ in case of suspension, but not the exact level of r.

6: This constraint is also a necessary condition for runs to occur. This is a reasonable description of actual banking procedures, as opposed to the operations of a mutual fund.

7: In that case. it is not optimal to decide at $t = 0$ that, in all states of the world, the asset will be liquidated at $t = 1$. If r_1 is the return on the asset at $t = 1$, with the square-root utility function one gets: $\sqrt{r_1+L_\upsilon} \leq \sqrt{r_1}+\sqrt{L_\upsilon} \leq \sqrt{r_1}+\rho E(\sqrt{r_2})$ for type 1. This also implies, for type 2, that: $\sqrt{r_1 + L_\upsilon} < \sqrt{r_1} + E(\sqrt{r_2})$. This generalizes easily to the case where $\alpha \neq 0$. Actually, condition (9) is slightly too restrictive. since risk-sharing between types ensures that the RHS is greater than the LHS.

8: More generally, $Q_{\bar{r}} = E((r_2 / r^+)^{1-a} | r_1 \geq \bar{r})$

9: Other types of normalization by a return in the support of r are possible, but only a normalization by r^+ is interpretable as a `share' of the maximal return. In addition, $pQ < 1$ is necessary to get equation (17) not binding. However $pQ < 1$ still obtains if the returns are normalized by $\ddot{r}$, s.t. $F\ddot{r}$ is sufficiently close to 1.

10: The other results derived in the paper are easily generalized to the case where a $\alpha \neq 0$. See in particular section 4.

11: An alternative assumption is that, when bankruptcy occurs at $t = 3n + 1$, failed intermediaries are banned from the mark-et and are not allowed to receive deposits from $t = 3n + 3$ to $t = 3n + 3f$.

12: See Vives(1991) for the existence of barriers to entry in banking.

13: For more details on interbank arrangements, see De Bandt(1994).

14: Timberlake(1984) also provides this quotation.

Appendix

A Equivalence between (P1) and (P2)

Notice that in (P1) and (P2) a slight complication comes from the fact that α enters the residual utility $U_0(\bar{r}\,.\ L_\upsilon,\ \alpha)$. To give a flavor of the results, let us assume, that it is not the case. For instance, let us assume that depositors get nothing in case of failure (U_0 = 0). As shown in A.1., there is a closed form solution to this program. However, this is no longer the case when U_0 takes the more general form assumed in section 3 of the paper: if α = 0, it is necessary to solve numerically one non-linear equation in λ_3, the Lagrange multiplier on the first incentive constraint; if α ≠ 0, a system of two non-linear equations in two unknowns, λ_3 and , α is to be solved.

A.1 $U_0 = 0, \alpha \neq 0$

Following Jacklin and Bhattacharya (1988), the solution to program (P1) is given by assuming the following sensible rule: taking the second incentive constraint, depositors will be allowed at $t = 2$ to withdraw funds in proportion to the return r_2. If $r_2 = r^+$, depositors will receive C_{2j}, otherwise they get $\tilde{c}_{2j} = c_{2j}\tilde{r}/r^+$, where $\tilde{r}_2$ and $\tilde{c}_{2j}$ are stochastic realizations from the point of view of $t = 1$ (Le. when r_1 is realized). This is program (P2). By showing that the two programs yield the saine solution, one proves that the suggested rule maximizes depositors utility.

The following lines therefore introduce, for a discrete support of $\tilde{r}_2 \in \{r_s, s.t. s = 1,...,n\}$,

$$E(\tilde{c}_{2j}) = E(c_{2js}) = \sum_s p_s\sqrt{c_{2js}} = Q\sqrt{c_{2j}}$$

Program (P2)

When α ≠ 0, (P2) is, for $\bar{r}$ given:

$$\max_{c_{ij},\alpha} P_{\bar{r}}\{p[U(c_{11}) + \rho U(c_{21})Q_{\bar{r}}] + (1-p)[U(c_{12}) + U(c_{22})Q_{\bar{r}}]\} \tag{1}$$

s.t.

$$\alpha + (1-\alpha)\bar{r} \geq pc_{11} + (1-p)c_{12} \tag{2}$$

$$(1-\alpha)r^+ \geq pc_{21} + (1-p)c_{22} \tag{3}$$

$$U(c_{11}) + \rho U(c_{21})Q_{\bar{r}} = U(c_{12}) + \rho U(c_{22})Q_{\bar{r}} \tag{4}$$

where the participation constraint as well as the residual utility in case of bankruptcy are omitted. Let $Q = Q_{\bar{r}}$ and λ_i, for i = 1, 2, 3, be the Lagrange multiplier on constraint (2), (3) and (4), respectively. The FOC's are the following, for 0 < α < 1:

$$p/\sqrt{c_{11}} - \lambda_1 p + \lambda_3/\sqrt{c_{11}} = 0 \tag{5}$$

$$p\rho Q/\sqrt{c_{21}} - \lambda_2 p + \lambda_3 \rho Q/\sqrt{c_{21}} = 0 \tag{6}$$

$$(1-p)/\sqrt{c_{12}} - \lambda_1(1-p) - \lambda_3/\sqrt{c_{12}} = 0 \quad (7)$$

$$(1-p)Q/\sqrt{c_{22}} - \lambda_2(1-p) - \lambda_3\rho Q/\sqrt{c_{22}} = 0 \quad (8)$$

$$\lambda_1(1-\bar{r}) = \lambda_2 r^+ \quad (9)$$

The CSC's are:

$$\lambda_1\left[\alpha + (1-\alpha)\bar{r} - pc_{11} - (1-p)c_{12}\right] = 0 \quad (10)$$

$$\lambda_2\left[(1-\alpha)r^+ - pc_{21} - (1-p)c_{22}\right] = 0 \quad (11)$$

Equations (5) and (6) can be written as: $(p+\lambda_3)/\sqrt{c_{11}} = \lambda_1 p$ and $\rho Q(p+\lambda_3)/\sqrt{c_{21}} = \lambda_2 p$

Using equation (9), for $\bar{r} < 1$, one gets[1]:

$$\sqrt{c_{21}} = \left(\frac{\rho Q r^+}{1-\bar{r}}\right)\sqrt{c_{11}} \quad (12)$$

$$= \rho\lambda\sqrt{c_{11}} \quad (13)$$

Similarly, $(1-p-\lambda_3)/\sqrt{c_{12}} = \lambda_1(1-p)$ and $Q(1-p-\lambda_3\rho)/\sqrt{c_{22}} = \lambda_2(1-p)$, with $\lambda_3 \le (1-p)/\rho$, which is required by λ_1 and λ_2 positive. This implies that:

$$\sqrt{c_{22}} = \frac{Qr^+(1-p-\lambda_3\rho)}{(1-\bar{r})(1-p-\lambda_3)}\sqrt{c_{12}} \quad (14)$$

$$= \eta(\lambda_3)\sqrt{c_{12}} \quad (15)$$

From equation (5) and (7), one also gets:

$$\sqrt{c_{12}} = \frac{(1-p-\lambda_3)p}{(p+\lambda_3)(1-p)}\sqrt{c_{11}} \quad (16)$$

$$= \varepsilon(\lambda_3)\sqrt{c_{11}} \quad (17)$$

Then plugging equations (13) and (15) into the incentive constraint (4) yields another relationship between c_{11}, and c_{l2}, namely:

$$\sqrt{c_{12}} = \frac{1+\gamma\rho^2 Q}{1+\rho Q\eta(\lambda_3)}\sqrt{c_{11}} \quad (18)$$

λ_3 is found by equating the LHS of (17) and (18). Let $1 + \gamma\rho^2 Q = A$ and $\frac{1-p}{p} = B$, so that:

$$\lambda_3 = \frac{\rho Q\gamma(1-p) - ABp + (1-p)}{AB + 1 + \rho^2 Q\gamma}$$

Or, equivalently,

$$\lambda_3 = p(1-p)\left(\frac{1+\rho Q\gamma}{1+\rho^2 Q\gamma} - 1\right) \quad (19)$$

[1]For $\bar{r} \ge 1$, the resource constraints imply that $\alpha = 0$, unless Q is too small so that the reservation constraint is not satisfied.

To determine α, use the resource constraints (2) and (3), as well as equations (13), (15), (17) to express the c_{ij}'s as functions of c_{11},. Therefore,

$$\frac{\alpha(1-\alpha)\bar{r}}{p+(1-p)\varepsilon(\lambda_3)^2} = \frac{(1-\alpha)r^+}{p\rho^2\gamma^2+(1-p)\eta(\lambda_3)^2\varepsilon(\lambda_3)^2}$$

Let the two denominators be D and E, respectively. It follows that:

$$\alpha = (1+\frac{E}{Dr^+ - E\bar{r}})^{-1} \qquad (20)$$

For $\alpha > 0$ to be true, I need $((D/Er^+ - \bar{r})^{-1} > -1$

From equations (17) and (19), it is clear -in the case where $\alpha > 0$- that $\frac{d\lambda_3}{d\bar{r}} > 0$ and $\frac{\varepsilon(\lambda_3)}{d\lambda_3} < 0$. In addition, $\lim_{\bar{r}\to 1}\gamma = \infty$, $\lim_{\bar{r}\to 1}\eta(\lambda_3) = \infty$ and $\lim_{\bar{r}\to 1} D/E = 0$. Hence, $\lim_{\bar{r}\to 1}(D/E)r^+ - \bar{r} = -\bar{r}$ and $\frac{-1}{\bar{r}} < -1$ for $0<\bar{r}<1$. Therefore, as $\bar{r}$ becomes close to 1, α reaches its lower bound.

Program (P1)

In the general case, for $\bar{r}$ given, the program to solve is:

$$\max_{c_{ij},\alpha} \quad P_{\bar{r}} \quad \{p[U(c_{11})+\rho E(U(\tilde{c}_{21})|r_1 \geq \bar{r}]+(1-p)[U(c_{12})+E(U(\tilde{c}_{22})|r_1 \geq \bar{r}]\} \qquad (21)$$

s.t.

$$\alpha+(1-\alpha)\bar{r} \geq pc_{11}+(1-p)c_{12} \qquad (22)$$

$$(1-\alpha)\bar{r}_2 \geq p\tilde{c}_{21}+(1-p)\tilde{c}_{22} \qquad (23)$$

$$U(c_{11})+\rho E(U(\tilde{c}_{21})|\ r_1 \geq \bar{r}) = U(c_{12})+\rho E(U(\tilde{c}_{22})|\ r_1 \geq \bar{r}) \qquad (24)$$

$\bar{r}_2$ is the random realization of the return on the risky asset at $t = 2$. There are as many constraints (23) as possible realizations.

As indicated before, the analysis focuses on the case whrere the distribution of returns is discrete and the'different realizations are indexed by s. The FOC's (5) and (7) are unchanged, with equations (6) and (8) replaced by equivalent equations where c_{2js} is the realization of $\tilde{c}_{2j}$ in state s:

$$p\rho p_s/\sqrt{c_{21s}} - \lambda_{2s}p + \lambda_3\rho p_s/\sqrt{c_{21s}} = 0 \qquad (25)$$

$$(1-p)p_s/\sqrt{c_{22s}} - \lambda_{2s}(1-p) - \lambda_3\rho p_s/\sqrt{c_{22s}} = 0 \qquad (26)$$

In addition, equation (9) is replaced by:

$$\lambda_1(1-\bar{r}) - \sum_s \lambda_{2s}r_s = 0 \qquad (27)$$

To prove equivalence between (Pl) and (P2), let us show that the FOC's are identical:

- Equation (17) is unchanged.
- The equivalent of equation (13) is derived from equation (5) and the new equation (25), namely $\lambda_1(1-\bar{r}) = \frac{p+\lambda_3}{p\sqrt{c_{11}}}(1-\bar{r})$ and $\Sigma_s\lambda_{2s}r_s = \Sigma_s \frac{\rho(p+\lambda_3)p_s r_s}{p\sqrt{c_{21s}}}$

Plugging into equation (27), this yields, after simplification:

$$\frac{1}{\sqrt{c_{11}}}=\frac{\rho}{1-\bar{r}}\sum_{s}\frac{r_s p_s}{\sqrt{c_{21s}}} \tag{28}$$

where p_s is the probability of state s.

In addition, from equations (25) and (26):

$$\sqrt{c_{22s}}=\frac{p}{1-p}\frac{1-p-\lambda_3\rho}{\rho(p+\lambda_3)}\sqrt{c_{21s}} \tag{29}$$

$$=\nu(\lambda_3)\sqrt{c_{21s}} \tag{30}$$

This condition holds, for all s, and in particular for $r_s = r^+$, so that: $(1 - \alpha)\, r_s = (p+(1-p)\nu(\lambda_3)^2)c_{21}$ and r_s/c_{21}, is constant for all s. Finally, since α is chosen *ex ante*:

$$\frac{r_s}{c_{21s}}=\frac{r^+}{c_{21}} \tag{31}$$

where c_{21} is the solution to (P1) when $r_s = r^+$. Thus, taking the square-root on both sides of (31) and multiplying by $\Sigma_s p_s\sqrt{r_s}$ yields $\Sigma_s r_s p_s/\sqrt{c_{21s}}=Qr^+/\sqrt{c_{21}}$ so that it is clear that equation (28) is equivalent to (13).

Similarly,

$$\left(\frac{1-p-\lambda_3}{1-p}\right)\frac{1-\bar{r}}{\sqrt{c_{12}}}=\sum_s r_s p_s\frac{1-p-\lambda_3\rho}{(1-p)\sqrt{c_{22s}}} \tag{32}$$

Using an equation similar to (31), but for c22, yields $\Sigma_s\frac{r_s p_s}{\sqrt{c_{22s}}}=\frac{Qr^+}{\sqrt{c_{22}}}$, so that equation (15) obtains.

Finally, using $c_{21s} = c_{21}r_s/r^+$ and $c_{22s} = c_{22}r_s/r^+$, it is clear that the objective function as well as the incentive constraints of the two programs (P1) and (P2) are the same. In addition, resource constraint (3) implies that equations (23) are satisfied for all s.

A.2 $U_0(\bar{r}, L_\upsilon, \alpha) \neq 0$

We know solve (P1) and (P2) in the general case assumed in section 3 of the paper.

A.2.1 $\alpha = 0$

Program (P2)

The program to solve, for $\bar{r}$ given, is:

$$\max_{c_{ij},\alpha} P_{\bar{r}}\left\{p\left[U(c_{11})+\rho U(c_{21})Q_{\bar{r}}\right]+(1-p)\left[U(c_{12})+U(c_{22})Q_{\bar{r}}\right]\right\} \tag{33}$$

s.t.

$$\bar{r}\geq pc_{11}+(1-p)c_{12} \tag{34}$$

$$r^+\geq pc_{21}+(1-p)c_{22} \tag{35}$$

$$U(c_{11})+\rho U(c_{21})Q_{\bar{r}}=U(c_{12})+\rho U(c_{22})Q_{\bar{r}} \tag{36}$$

The residual utility $U_0(\bar{r}, L_\upsilon, \alpha)$ is now fixed, silice the parameters $\bar{r}$, U_0 and $\alpha = 0$ are given. It is therefore omitted in the objective function. The FOC's are the same as in A.1.1. The only difference is that, since α is no longer a choice variable, equation (9) is not relevant.

On the one hand, one gets equation (17) from equations (5) and (7). Equations (6) and (8) yièld, on the other hand:

$$\sqrt{c_{22}} = \nu(\lambda_3)\sqrt{c_{21}} \quad (37)$$

Plugging these two equations into the these constraints yields: $c_{11} = \frac{\bar{r}}{p+(1-p)\varepsilon(\lambda_3)^2}$ and $c_{21} = \frac{r^+}{p+(1-p)\nu(\lambda_3)^2}$

Finally, λ_3 is determined by the following equation:

$$(1-\varepsilon(\lambda_3))\sqrt{\frac{\bar{r}}{p+(1-p)\varepsilon(\lambda_3)^2}} = (\nu(\lambda_3)-1)\rho Q\sqrt{\frac{r^+}{p+(1-p)\nu(\lambda_3)^2}} \quad (38)$$

It is shown in appendix B that there is only one real root to equation (38).

Program (P1)

For $\bar{r}$ given, the program to solve is:

$$\max_{c_{ij}} P_{\bar{r}}\left\{p\left[U(c_{11}) + \rho E(U(\tilde{c}_{21})|r_1 \geq \bar{r})\right] + (1-p)\left[U(c_{12}) + E(U(\tilde{c}_{22})|r_1 \geq \bar{r})\right]\right\} \quad (39)$$

s.t.

$$\bar{r} \geq pc_{11} + (1-p)c_{12} \quad (40)$$

$$r^+ \geq p\tilde{c}_{21} + (1-p)\tilde{c}_{22} \quad (41)$$

$$U(c_{11}) + \rho E(U(\tilde{c}_{21})|r_1 \geq \bar{r}) = U(c_{12}) + \rho E(U(c_{22})|r_1 \geq \bar{r}) \quad (42)$$

where there are as many constraints (41) as possible realizations of the risky return at $t = 2$. Again, consomption at $t = 2$ is written $\tilde{c}_{2j}$.

Equation (15) and (30) still obtain. In the latter case, it holds for all state s, hence, in particular for $r_s = r^+$ so that equation (37) is also true.

Plugging back into the there constraints, yields:

$$(1-\varepsilon(\lambda_3))\sqrt{\frac{\bar{r}}{p+(1-p)\varepsilon(\lambda_3)^2}} = (\nu(\lambda_3)-1)\rho\sum_s p_s\sqrt{\frac{r^+}{p+(1-p)\nu(\lambda_3)^2}} \quad (43)$$

If $Q = \Sigma_s p_s\sqrt{r_s / r^+}$ equation (43) and (38) are totally equivalent.

To sum up, like in Al, applying the liquidation rule $c_{21s} = c_{21}r_s/r^+$ and $c_{22s} = c_{22}\, r_s/r^+$, the solutions to (P1) and (P2) are equivalent.

A.2.2 $\alpha \neq 0$

In that case, equation (9) becomes

$$E(\left[1-r-L_\upsilon\right]U'(\alpha+\left[1-\alpha\right]\left[r+L_\upsilon\right])|r<\bar{r})\Pr\{r<\bar{r}\} + \lambda_1(1-\bar{r}) = \lambda_2 r^+ \quad (44)$$

where $U'(.)$ is the first derivative of U(.) = √.

From the FOC's. equations (17) and (37) still hold. Plugging them into the resource constraint ields $c_{11} = \frac{\alpha+(1-\alpha)\bar{r}}{p+(1-p)\varepsilon(\lambda_3)^2}$ and $c_{21} = \frac{(1-\alpha)r^+}{p+(1-p)\nu(\lambda_3)^2}$.

Hence (38) becomes the following non-linear equation in (λ_3, α)

$$(1-\varepsilon(\lambda_3))\sqrt{\frac{\alpha+(1-\alpha)\bar{r}}{p+(1-p)\varepsilon(\lambda_3)^2}} = (\nu(\lambda_3)-1)\rho Q\sqrt{\frac{(1-\alpha)r^+}{p+(1-p)\nu(\lambda_3)^2}} \tag{45}$$

In addition, from the first two FOC's (5) and (6) :

$$\lambda_1 = \frac{(p+\lambda_3)\sqrt{p+\varepsilon(\lambda_3)^2(1-p)}}{p\sqrt{\alpha+(1-\alpha)\bar{r}}} \tag{46}$$

$$\lambda_2 = \frac{(p+\lambda_3)\sqrt{p+\nu(\lambda_3)^2(1-p)}}{p\sqrt{(1-\alpha)r^+}} \tag{47}$$

Plugging (46) and (47) in (44) yields another non-linear equation in (λ_3, α).

The solution to (P2) is therefore given by this system of two non-linear equations in (λ_3, α). For α given, there is a unique solution to (45), as shown in Appendix B. Considering equation (44) - for λ_3 given and $\bar{r} < 1$- notice that λ_1 is decreasing in α, whereas λ_2 is increasing. In addition the terni related to residual utilityin (44) is decreasing in α, for $\bar{r}$ not too large. Hence, unicity of the solution of (44) in α is very likely to hold. This conclusion is buttressed by comparative statics analysis on equation (44) : an increase in $\bar{r}$ induces an increase λ_1 and a decrease in λ_2. Neglecting the terni involving the residual utility -which is of second order in comparison to the other terms- the new equilibrium implies a decrease in α.

The proof of the equivalence between (Pl) and (P2) is identical to the preceding cases.

B Solution to program (P2) in terms of λ_3

B.1 Unicity

In Appendix A, it is shown that, in the case where α = 0

$$\sqrt{c_{12}} = \frac{(1-p-\lambda_3)p}{(p+\lambda_3)(1-p)}\sqrt{c_{11}} \tag{48}$$

$$= \varepsilon(\lambda_3)\sqrt{c_{11}} \tag{49}$$

$$\sqrt{c_{22s}} = \frac{1-p-\lambda_3\rho}{\rho(p+\lambda_3)}\sqrt{c_{21s}} \tag{50}$$

$$= \nu(\lambda_3)\sqrt{c_{21s}} \tag{51}$$

Where it was assumed that positivity constraints on the c_{ij}'s were not binding, namely that $\lambda_3 < 1 - p$. Plugging these two equations into the incentive constraint yields, using $Q_{\bar{r}} = Q$ to simplify notations

$$(1-\varepsilon(\lambda_3))\sqrt{c_{11}} = \rho Q(\nu(\lambda_3)-1)\sqrt{c_{21}} \tag{52}$$

The LHS simplifies in $\lambda_3\sqrt{c_{11}}/(p+\lambda_3)(1-p)$ and the RHS in $\frac{Q((1-p)p+\rho(p^2-p-\lambda_3))}{(p+\lambda_3)(1-p)}$ Finally:

$$\sqrt{\frac{c_{11}}{c_{21}}} = \frac{Q}{\lambda_3}((1-p)p+\rho(p^2-p-\lambda_3)) \tag{53}$$

for $(1-p)p+\rho(p^2-p-\lambda_3)>0$, or, equivalently, for $\lambda_3 < \frac{(1-p)p(1-\rho)}{\rho}$

Plugging equations 49 and 51 into the resource constraints yields : $D\bar{r} = (p+(1-p)\varepsilon(\lambda_3)^2)c_{11}$ and $r^+ = (p+(1-p)\nu(\lambda_3)^2)c_{21}$. Finally:

$$\sqrt{\frac{c_{11}}{c_{21}}} = \sqrt{\frac{\bar{r}(p+(1-p)\nu(\lambda_3)^2}{r^+(p+(1-p)\varepsilon(\lambda_3)^2}} \tag{54}$$

Hence λ_3 is a solution to:

$$\frac{Q}{\lambda_3}(1-p)p+\rho(p^2-p-\lambda_3) = \sqrt{\frac{\bar{r}(p+(1-p)\nu(\lambda_3)^2)}{r^+(p+(1-p)\varepsilon(\lambda_3)^2)}} \tag{55}$$

To argue that there exists only one real root, it is convenient to notice that the LHS of equation (55) is a monotone decreasing function of λ_3:

$$\frac{dLHS}{d\lambda_3} = \frac{-\rho Q}{\lambda_3^2} < 0 \tag{56}$$

Concerning the RHS, notice that:

$$\frac{d\nu(\lambda_3)}{d\lambda_3} = \frac{p+(1-p)/\rho}{(p+\lambda_3)^2} < 0 \tag{57}$$

$$\frac{d\varepsilon(\lambda_3)}{d\lambda_3} = \frac{1}{(p+\lambda_3)^2} < 0 \tag{58}$$

But to assess the relative effect of these two variables on the RHS of equation (55), it is necessary to compute the derivative of the whole expression.

Straightforward expansion of the square of the RHS of equation *55* yields

$$RHS = \left(\frac{\bar{r}}{r^+}\right)\left(\frac{\lambda_3^2\rho^2(1-p)p+2\lambda_3^2p^2(1-p)^2(\rho^2-\rho)+p^2(1-p)^2(\rho^2p+(1-p))}{\rho^2(\lambda_3^2p(1-p)+p^2(1-p)^2)}\right)$$

which can be written as

$$RHS = \frac{\bar{r}}{r^+}\frac{A\lambda_3^2+2\lambda_3+C}{\rho^2(D\lambda_3^2+E)}$$

Hence

$$\frac{dRHS}{d\lambda_3} = \left(\frac{\bar{r}}{r^+\rho^2}\right)\left(\frac{(2A\lambda_3+2B)(D\lambda_3^2+E)-2D\lambda_3(A\lambda_3^2+2B\lambda_3+C)}{(D\lambda_3^2+E)^2}\right)$$

The numerator is $-2(BD\lambda_3^2+(CD-AE)\lambda_3-BE)$. The roots of this equation are:

$$\lambda_3 = \frac{(AE - CD) \pm \sqrt{\Delta}}{2BD}$$

With $\sqrt{\Delta} = p^3(1-p)^3\sqrt{(1-p)^2(\rho^2-1)^2 + 4p(\rho^2-\rho)^2}$

Therefore the derivative is equal to zero for the two solutions:

$$\lambda_3' = \frac{(1-p)(\rho^2-1) + \sqrt{(1-p)^2(\rho^2-1)^2 + 4p(\rho^2-\rho)^2}}{2(\rho^2-\rho)}$$

$$\lambda_3'' = \frac{(1-p)(\rho^2-1) + \sqrt{(1-p)^2(\rho^2-1)^2 + 4p(\rho^2-\rho)^2}}{2(\rho^2-\rho)}$$

Where $\lambda_3' < 0$ and $\lambda_3'' > 0$

Since $BD = p^3(1-p)^3(p^2 - p) < 0$, the RHS of equation 55 is decreasing between λ_3' and λ_3'' and increasing elsewhere.

In addition, $\lim_{\lambda_3 \to +\infty} RHS = \bar{r}/r^+ > 0$ and $\lim_{\lambda_3 \to 0} RHS = \frac{\bar{r}}{r^+}(p + \frac{1-p}{\rho^2})$, so that it has a finite limit when λ_3 tends to zero as long as ρ is bounded away from zero.

Finally, from equation (55), λ_3 is determined by the intersection between:

- A convex curve, which tends to $+\infty$ as λ_3 approaches zero, and becomes negative for $\lambda_3 > \frac{(1-p)p(1-\rho)}{\rho}$ then converges to $-\rho Q$. From (53); only the first part is relevant.
- Another decreasing curve, RHS, bounded away from zero, unless $\bar{r} = 0$.

The intersection is unique, since the slope of LHS is steeper than the one of RHS (for λ_3 small). This is clear from the graph presented in De Bandt(1994). $Q < 1$ so that the LHS of equation (55) is shifted downward and to the left and the RHS, which is almost horizontal, is shifted upward with $\bar{r}$.

B.2 Comparative statics

An increase in $\bar{r}$ shifts unambiguously the LHS and the RHS of equation (55). The final effect on λ_3 depends on the relative change of the two curves.

Equation (55) can be written as:

$$A(\lambda_3)Q - \left[B(\lambda_3)\bar{r}\right]^{1/2} = 0 \qquad (59)$$

Totally diferentiating (59) yields:

$$\frac{d\lambda_3}{d\bar{r}} = -\frac{\frac{\partial A(\lambda_3)Q}{\partial \bar{r}} - \frac{\partial\left[B(\lambda_3)\bar{r}\right]^{1/2}}{\partial \bar{r}}}{\frac{\partial A(\lambda_3)Q}{\partial \lambda_3} - \frac{\partial\left[B(\lambda_3)\bar{r}\right]^{1/2}}{\partial \lambda_3}} \qquad (60)$$

The denominator of (60) is negative, as we saw above. Let us look for the conditions under which the numerator is negative too, so that $d\lambda_3 / d\bar{r} \le 0$.

$$\frac{\partial A(\lambda_3)Q}{\partial \bar{r}} - \frac{\partial\left[B(\lambda_3)\bar{r}\right]^{1/2}}{\partial \bar{r}} = A(\lambda_3)\frac{\partial Q}{\partial \bar{r}} - \frac{1}{2}B(\lambda_3)^{1/2}(\bar{r})^{-1/2} \qquad (61)$$

Using (59), (61) is negative iff:

$$\frac{\bar{r}}{Q}\frac{\partial Q}{\partial \bar{r}} \leq 1/2 \tag{62}$$

Since $Q \in \left[(\Sigma_G + \Sigma_B)/2; \Sigma_G\right]$ (and recall that $\Sigma_A \leq 1$, for A = G, B) for $\bar{r}$ chosen on the whole domain of r, this condition is very likely to hold, especially if the distribution of returns is sufficiently smooth and the variance of the "good" and the "bad" distributions are close to each other. The discrete approximation of (62) is always satisfied in the numerical simulations provided here.

Hence an increase in $\bar{r}$ implies a decrease in λ_3.

Another property of the solution is that the change in λ_3 is usually very small in response to a change in $\bar{r}$. Usually the solution is $\lambda_3 \approx 0$. In that case, $\varepsilon\ \lambda_3 \approx 1$, $\nu\ \lambda_3 \approx 1/\rho$

Therefore

$$\frac{d}{d\bar{r}}\left(\frac{\bar{r}}{p + (1-p)\varepsilon(\lambda_3)}\right) > 0 \tag{63}$$

$$\frac{d}{d\bar{r}}\left(\frac{E\sqrt{r_2 | r_1 \geq \bar{r})}}{p + (1-p)\nu(\lambda_3)}\right) > 0 \tag{64}$$

B.3 Proof of Proposition 1

According to Lemma 1, Σ_G and Σ_B large imply that the distribution of r dominates the safe asset, so that $\alpha = 0$. In that case, expressing the c_{ij}'s as a function of c_{11} and plugging into the objective flinction yields: $c_{11} = \frac{\bar{r}}{p+(1-p)\varepsilon(\lambda)^2}$ and $c_{21} = \frac{r^+}{p+(1-p)\nu(\lambda)^2}$ so that:

$$U(\bar{r}) = P_{\bar{r}}\left\{\sqrt{\frac{\bar{r}}{p+(1-p)\varepsilon(\lambda)^2}}\left[p + (1-p)\varepsilon(\lambda)\right] + Q_{\bar{r}}\sqrt{\frac{r^+}{p+(1-p)\nu(\lambda)^2}}\left[p\rho + (1-p)\nu(\lambda))\right]\right\} + U_0(\bar{r}, L_\upsilon)$$

where $Q_{\bar{r}}\sqrt{r^+} = E(\sqrt{r} | r_1 \geq \bar{r})$.

It is useful to notice that $Q_{\bar{r}}$ is increasing in $\bar{r}$, so that , when $\bar{r}$ is increased $\theta_{\bar{r}}$ increases too, since the first period resource constraint is relaxed.

This is confirmed by the following differentials, derived in Appendix B.2:

$\frac{d\lambda}{d\bar{r}} \leq 0$, $\frac{d\varepsilon(\lambda)}{d\lambda} \leq 0$, and $\frac{d\nu(\lambda)}{d\lambda} \leq 0$.

In addition, $\frac{d}{d\bar{r}}\left(\frac{\bar{r}}{p+(1-p)\varepsilon(\lambda)^2}\right) \geq 0$ and $\frac{d}{d\bar{r}}\left(\frac{E(\sqrt{r}|r_1 \geq \bar{r})}{p+(1-p)\nu(\lambda)^2}\right) \geq 0$.

As long as $P_{\bar{r}}$ does not decrease too rapidly with $\bar{r}$ in the lover range of r, $P_{\bar{r}}\theta_{\bar{r}}$ is increasing. But $P_{\bar{r}}$ tends to zero as $\bar{r}$ approaches r^+ and the function $P_{\bar{r}}\theta_{\bar{r}}$ ends up being decreasing in the upper range of r. The extra term, U_0 ($\bar{r}$, L_υ) is increasing in $\bar{r}$ and counterbalances the reduction in utility caused by $P_{\bar{r}}$ when $\bar{r}$ becomes large. If L_υ is small, $U(\bar{r})$ may even be decreasing from the lover range of r and in the case where $U(\bar{r}) < 1$ with $\alpha = 0$, the solution has to include $\alpha > 0$, since $U(\bar{r}) \geq 1$ with $\alpha = 1$.

B.4 Extension to the solution of (P3)

As indicated in 5.1, (P3) is solved by looking for the optimal $\alpha(\bar{r}$ such that depositors receive their reservation utility and the utility of the intermediary is maximized.

For $\bar{r}$ and α given we are back to problem (P2), where depositor's utility is maximized under resource and incentive constraints. Using the saine FOC's, I can write the resource constraints as:

$$\alpha + (1-\alpha)\bar{r} = (p + (1-p)\varepsilon(\lambda_3)^2)c_{11}$$

$$(1-\alpha)\bar{r} = (p + (1-p)\nu(\lambda_3)^2)(\frac{1-\varepsilon(\lambda_3)}{\rho Q(\nu(\lambda_3)-1)})c_{11}$$

Equating these two equations allows to solve for λ_3 . Imposing $\nu(\lambda_3) \geq 1$ ensures that the solution is unique.

C Proof of Lemma 2

The strategy to solve (P3) in the case $\alpha(\bar{r}) \neq 0$ is inspired by the solution to (P2):

1. For $\bar{r}$ given, look for the optimal share of the endowment invested in the safe asset $\alpha(\bar{r})$ that yields the depositor's reservation utility of 1. The smallest such $\alpha(\bar{r})$ solves (P3) for $\bar{r}$ given.

2. The solution of (P3) is therefore given by $\bar{r}^* = \arg\max_{\bar{r}} (1-\alpha(\bar{r}))E(r-\bar{r})$

In the case where $\bar{r} = 0$, the two resource constraints are: $\alpha = pc_{11} + (1-p)c_{12}$ and $(1-\alpha)r^+ = pc_{21} + (1-p)c_{22}$. Let us define the equilibrium value of α as $\alpha(0)$. If $r = \bar{r} > 0$, the resource constraints are: $\alpha + (1-\alpha)\bar{r} = pc_{11} + (1-p)c_{12}$ and $(1-\alpha)r^+ = pc_{21} + (1-p)c_{22}$, with the equilibrium value of α noted $\alpha(\bar{r})$. For α given, the first resource constraint is relaxed when $\bar{r}$ becomes strictly positive. For $\bar{r}$ small, $prob\{r < \bar{r}\} \approx 0$, so that depositor's utility is higher for $\bar{r} > 0$. But, under monopoly, depositors only receive their reservation utility, so that $\alpha(0)\ \alpha \geq (\bar{r})$: the equilibrium share of the endowment invested in the safe asset is higher with $\bar{r} = 0$. From the monopolist intermediary's point of view, numerical simulations presented in De Bandt(1994) show that: $(1-\alpha(0))E(r) \geq (1-\alpha(\bar{r}))E(r-\bar{r})$. Notice that $\bar{r} = 0$ reallocates the endowment between a risk neutral intermediary and a risk averse depositor. The gain for the depositor are small in comparison to the loss by the intermediary. Independently of the introduction of a small risk of failure, the relaxation of the first resource constraint is partially offset by a tightening of the second resource constraint ($\alpha(0) \geq \alpha(\bar{r}) + (1-\alpha(\bar{r}))\bar{r}$, for $\bar{r}$ small). Hence $\bar{r} > 0$ may introduce a suboptimal intertemporal allocation of consumption.

References

[1] Aghion, A., P. Bolton and M. Dewatripont (1988), `Interbank lending and contagious bank runs', mimeo, MIT.

[2] Aglietta, M. and Ph. Moutot (1993), `L'evolution des structures financières des grands pays et la prevention du risque de système dans l'U.E.M.' *Cahiers economiques et monétaires, 41.*

[3] Archibald, G. C. (1992), *Information, incentives and the economics of control,* Cambridge University Press.

[4] Bernanke, B. S. and M. Gertler (1987), `Banking and macroeconomic equilibrium' in *New appoach to monetary economics, W. A.* Barnett and K. J. Singleton editors, Cambridge University Press, 89-111.

[5] Bhattacharya, S. and D. M. Gale (1987), `Preference shocks, liquidity and central bank policy' in *New appoach to monetary economics, W. A.* Barnett and K. J. Singleton editors, Cambridge University Press, 69-88.

[6] Boyd, J. H. and E. C. Prescott (1986), `Financial intermediary-coalitions' *Journal of Economic Theory, 38,* 211-232.

[7] Calomiris, C. W. and C. M. Kahn (1991), `The role of demandable debt in structuring optimal banking arrangements', *American Economic Review, vol.* 81, no 3, 497-513.

[8] Calomiris, C. W. and G. Gorton (1991), `The origin of banking panics: models, facts and bank regulation' in *Financial markets and finacial crisis, R. G.* Hubbard ed., Chicago University Press.

[9] Cannon, J.G. (1910), *Clearing Houses,* Nat. Money. Comm., 61st Congress, U.S. Senate, Government Printing Office.

[10] Coate, S. and M. Ravaillion (1993), `Reciprocity without commitment, characterization and performance of informal insurance arrangements' *Journal of Development Econ., 40,* 1-24.

[11] De Bandt, 0. (1994), `Competition among financial intermediaries and the risk of contagious failures' Pli. D. *dissertation,* University of Chicago, Department of Economics.

[12] Diamond, D. W. and P. H. Dybvig (1983), `Bank runs, deposit insurance and liquidity' *Journal of Political Economy,* 91(3), 401-419.

[13] Economides, E., R. G. Hubbard and D. Palia (1994), `The political economy of branching restrictions and deposit insurance: a model of monopolistic competition among small and large banks', mimeo.

[14] Friedman, M. and A. Schwarz (1963), A *monetary History of the United States: 1867-1960,* Princeton University Press.

[15] Gendreau, B. C. and S. S. Prince (1986), `The cost of bank failures: historical evidence', *Federal Reserve Bank of Philadelphia Business review.*

[16] Gorton, G. (1985), `Bank suspension of convertibility' *Journal of Monetary Economic,* 15, 177-193.

[17] Gorton, G. and J. D. Mullineaux (1987), `The joint production of confidence: endogeneous regulation and nineteenth century commercial bank clearing houses' *Journal of Money, Credit and Banking,* 19(4).

[18] Jacklin, C. and S. Bhattacharya (1988), `Distinguishing Panics and Information-based Ban], runs: Welfare.and.Policy implications' *Journal of Political Economy,* 96(3).

[19] Krasa, S. and A. P. Villamil (1992), `The optimal size of banks', Oxford *Economic papers,* 44, 725-749.

[20] Matutes, C. and a. Vives (1992), Competition for deposits, risk of failure, and regulation in banking', mimeo, University of Barcelona.

[21] Mishkin, F. S.(1992), *The economics of money, 6anking and financial markets.*

[22] Mishkin, F. S. (1991), Asymmetric information and financial crises: an historical perspective', in *Financial markets and financial crises, R. G.* Hubbard, editor, Chicago University Press.

[23] Paroush (1988), J., `The domino effect and the supervision of the banking system' *The Journal of Finance,* 43(5), 1207-18.

[24] Smith, B. D. (1991), `Bank panics, Suspensions and geography: some notes on the contagion of fear in banking' *Economic inquiry,* 29(2), 230-248.

[25] Sprague, 0. M. W. (1910), *History of crises under the National Banking system,* National Monetary Commission, 61st Congress, U.S. Senate, Government Printing Office.

[26] Terlizzese, D. (1989), On incentive-compatible sharing contracts', *Banca d'Italia Temi di Discussione, 20.*

[27] Timberlake, R. H., Jr (1984), The central bank role of clearing houses associations', *Journal of Money, Credit and Banking, 16(1), 1-15.*

[28] Vives, X. (1991), Banking competition and European integration', in *European financial integration, A.* Giovannini and C. Mayer editors, Cambridge University Press.

[29] Williamson, S. D. (1989), Restrictions on financial intermediaries and implications for aggregate fluctuations: Canada and the United States 1870-1913', N.B.E.R. Macroeconomics *Annual.*

Notes d'Études et de Recherche

1. C. Huang and H. Pagès, "Optimal Consumption and Portfolio Policies with an Infinite Horizon: Existence and Convergence," May 1990.

2. C. Bordes, « Variabilité de la vitesse et volatilité de la croissance monétaire : le cas français », février 1989.

3. C. Bordes, M. Driscoll and A. Sauviat, "Interpreting the Money-Output Correlation: Money-Real or Real-Real?," May 1989.

4. C. Bordes, D. Goyeau et A. Sauviat, « Taux d'intérêt, marge et rentabilité bancaires : le cas des pays de l'OCDE », mai 1989.

5. B. Bensaid, S. Federbusch et R. Gary-Bobo, « Sur quelques propriétés stratégiques de l'intéressement des salariés dans l'industrie », juin 1989.

6. O. De Bandt, « L'identification des chocs monétaires et financiers en France : une étude empirique », juin 1990.

7. M. Boutillier et S. Dérangère, « Le taux de crédit accordé aux entreprises françaises : coûts opératoires des banques et prime de risque de défaut », juin 1990.

8. M. Boutillier and B. Cabrillac, "Foreign Exchange Markets: Efficiency and Hierarchy," October 1990.

9. O. De Bandt et P. Jacquinot, « Les choix de financement des entreprises en France : une modélisation économétrique », octobre 1990 (English version also available on request).

10. B. Bensaid and R. Gary-Bobo, "On Renegotiation of Profit-Sharing Contracts in Industry," July 1989 (English version of NER n° 5).

11. P. G. Garella and Y. Richelle, "Cartel Formation and the Selection of Firms," December 1990.

12. H. Pagès and H. He, "Consumption and Portfolio Decisions with Labor Income and Borrowing Constraints," August 1990.

13. P. Sicsic, « Le franc Poincaré a-t-il été délibérément sous-évalué ? », octobre 1991.

14. B. Bensaid and R. Gary-Bobo, "On the Commitment Value of Contracts under Renegotiation Constraints," January 1990 revised November 1990.

15. B. Bensaid, J.-P. Lesne, H. Pagès and J. Scheinkman, "Derivative Asset Pricing with Transaction Costs," May 1991 revised November 1991.

16. C. Monticelli and M.-O. Strauss-Kahn, "European Integration and the Demand for Broad Money," December 1991.

17. J. Henry and M. Phelipot, "The High and Low-Risk Asset Demand of French Households: A Multivariate Analysis," November 1991 revised June 1992.

18. B. Bensaid and P. Garella, "Financing Takeovers under Asymetric Information," September 1992.

19. A. de Palma and M. Uctum, “Financial Intermediation under Financial Integration and Deregulation,” September 1992.

20. A. de Palma, L. Leruth and P. Régibeau, “Partial Compatibility with Network Externalities and Double Purchase,” August 1992.

21. A. Frachot, D. Janci and V. Lacoste, “Factor Analysis of the Term Structure: a Probabilistic Approach,” November 1992.

22. P. Sicsic et B. Villeneuve, « L'afflux d'or en France de 1928 à 1934 », janvier 1993.

23. M. Jeanblanc-Picqué and R. Avesani, “Impulse Control Method and Exchange Rate,” September 1993.

24. A. Frachot and J.-P. Lesne, “Expectations Hypothesis and Stochastic Volatilities,” July 1993 revised September 1993.

25. B. Bensaid and A. de Palma, “Spatial Multiproduct Oligopoly,” February 1993 revised October 1994.

26. A. de Palma and R. Gary-Bobo, “Credit Contraction in a Model of the Banking Industry,” October 1994.

27. P. Jacquinot et F. Mihoubi, « Dynamique et hétérogénéité de l'emploi en déséquilibre », septembre 1995.

28. G. Salmat, « Le retournement conjoncturel de 1992 et 1993 en France : une modélisation VAR », octobre 1994.

29. J. Henry and J. Weidmann, “Asymmetry in the EMS Revisited: Evidence from the Causality Analysis of Daily Eurorates,” February 1994 revised October 1994.

30. O. De Bandt, “Competition Among Financial Intermediaries and the Risk of Contagious Failures,” September 1994 revised January 1995.

Pour tous commentaires ou demandes sur les Notes d'Études et de Recherche, contacter la bibliothèque du Centre de recherche à l'adresse suivante :

For any comment or enquiries on the Notes d'Études et de Recherche, contact the library of the Centre de recherche at the following address :

BANQUE DE FRANCE
41-1391 - Centre de recherche
75049 Paris Cedex 01
tél : (0)1 42 92 49 55
fax : (0)1 42 92 62 92
email : thierry.demoulin@banque-france.fr